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GÖDEL, KAPLOW, SHAVELL:
CONSISTENCY AND COMPLETENESS IN SOCIAL DECISIONMAKING

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We hear within ourselves the constant cry: There is the problem, seek the solution. You can find it through pure thought.

David Hilbert, The Problems of Mathematics

Lo, this only have I found,
that God hath made man upright;
but they have sought out many inventions

Ecclesiastes 7, 29

Abstract

The recent debate on what criteria ought to guide social decisionmaking has focused on consistency: it has been argued that criteria contradicting one another – namely, welfare and fairness – should not be simultaneously employed in order for policy assessment to be consistent. In this article, I raise the related problem of completeness – that is, the question of whether or not a set of consistent criteria is capable of providing answers to *all* social decision problems. If not, as I suggest might be the case, then the only way to decide otherwise undecidable issues is to simultaneously employ both welfare and fairness, which implies a certain degree of inconsistency within the system.

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* *Email*: g.darimattiacci@econ.uu.nl; *SSRN author page*: http://ssrn.com/author=333631. The title of this article paraphrases *Gödel, Escher, Bach: An Eternal Golden Braid* (Hofstadter 1999), which provided the initial inspiration for my research. It also acknowledges the intellectual reference points of my analysis, which rest on Kaplow and Shavell (2002) and Gödel (1931). Although all errors remain mine, I wish to thank Johan van Benthem, Tina Heubeck, Lewis Kornhauser, Henrik Lando, Chiseche Mibenge, Hans-Bernd Schäfer and the participants in the Law and Economics workshop of the 2003 IVR World Legal Philosophy Conference in Lund for helpful suggestions. Devlin Cooper provided valuable editorial assistance. In this article, in order to provide the reader with information on the period in which certain discoveries in logic and mathematics occurred, I indicate between square brackets the dates of birth (and death, if the case) of those scholars who contributed to them.

1. Introduction

Fairness versus Welfare (Kaplow and Shavell 2002, hereinafter FVW),[1] has recently revived scholarly attention to the vexed question of what value(s) should guide legislators, regulators, judges, academics, and other individuals who shape policies affecting society at large. For example, how does one decide whether an injurer should pay damages to the victim of an accident, under what conditions promises should be fulfilled, or whether a subsidy should be given to single mothers?

FVW claims that these kinds of policies should be *solely* evaluated according to the effects they have on the well-being of individuals – the welfarist criterion – and, thus, *no independent weight* should be given to notions of fairness – a catchall term for any non-welfarist criterion. The crux of the argument is that pursuing ideals of fairness might actually reduce the well-being of some or all individuals in society, which is not a desirable effect.[2] While taking no stand concerning the normative component of the analysis (whether or not it is socially desirable to maximize individuals' well-being), I will analyze this claim from a logical perspective.

The conclusions reached in FVW rest on premises that are admittedly[3] non-falsifiable: a value judgment on the aim of social policy (that the well-being of individuals should be maximized) and a tautological definition of the problem (the welfarist criterion maximizes the well-being of individuals, while any non-welfarist fairness-based criterion does not). On this basis, the argument cannot be wrong, but is it right?

I will argue that FVW should be narrowly interpreted as calling for consistency in social decisionmaking: FVW rejects the use of criteria (whatever they might be) that yield conflicting policy decisions. This reinterpreted FVW only proves that inconsistent criteria yield incompatible outcomes but does not provide guidelines for the use of one or another criterion. It only contends

[1] The book has been accompanied by a series of articles dealing with the conflict between fairness and welfare (Kaplow and Shavell 1999), formalizing that conflict (Kaplow and Shavell 2001) or responding to criticisms (Kaplow and Shavell 2000 and 2003) advanced in Chang (2000a and 2000b), Craswell (2003), Kornhauser (2003) and Waldron (2003). For additional replies to FVW, see Dorff (2002), Fransworth (2002), Fried (2002), and McDonnel (2003).

[2] In FVW, two additional arguments support the analysis. One is that notions of fairness lack precise definitions and rationales and are generally non-consequentialist (FVW, pp. 45-51, 59-62). I see this claim as a part of what I discuss in section 4 as 'a call for consistency'. The other acknowledges that language phrased in terms of fairness might actually hide a welfarist methodology at times. In this case, notions of fairness are not being given independent account as they simply function as proxies for the welfarist criterion, and, hence, their use is admitted under FVW, (pp. 69-81, 381-402). See also Kaplow and Shavell (2003) reinforcing the three claims.

[3] See sections 3.2 and 3.3.

that fairness and welfare should not be simultaneously employed, but it leaves open the question of which to use.

I will then raise a problem that goes beyond the reach of FVW. FVW describes a consistent, formal system for policy assessment but does not discuss the question of whether such a system will in fact be able to treat *all* social decisions. There might in fact exist some policy issues for which the system is unable to provide a solution. In other words, the question is whether a consistent system for social decisionmaking is complete.

Results attained in the field of mathematical logic seem to give a negative answer or at least call for some alertness. Gödel (1931)[4] proved that formal axiomatic systems of sufficient complexity are inherently incomplete. That is, there are problems for which a solution exists but cannot be found by use of the system. If this were also true for the system described in FVW, there would arise an immediate policy consequence. Namely, if a consistent system is not complete, the only way to solve issues that would remain otherwise unanswered is to accept some inconsistency. In FVW, this would mean giving independent weight to some of those non-welfarist criteria that were rejected at the outset precisely due to inconsistencies with welfare economics.

A paradox then emerges. For the sake of consistency, social decisionmaking should be based either on welfare or on fairness but not on both, but for the sake of completeness, it might be necessary for one criterion to complement the other. Gödel in fact showed that completeness and consistency cannot be simultaneously attained. A related problem concerns the identification of those issues that the system cannot resolve, and it turns out that unsolvable problems will have to be identified one by one since ex ante it is impossible to enumerate all of them in a systematic way (Church 1936).

In addition, Gödel also proved that consistency cannot be rigorously proved within a formal system. Thus, the absence of inconsistencies cannot be formally established within FVW itself even if FVW struggles to avoid inconsistencies by eliminating conflicting criteria of choice. After examining the logic of FVW (sections 2 and 3), I will address the problem of consistency in social decisionmaking (in section 4). I will then relate this to the logic literature on the completeness of

[4] Kurt Gödel [1906-1978], an Austrian-born logician, studied and produced the results mentioned in this article at the University of Vienna. His 1931 article gained him the *Habilitation* and a position as *Privatdozent* in Vienna at the age of 25, a year after receiving his doctorate in mathematics on the completeness of first-order logic. From 1938, he was a permanent member of the Center for Advanced Studies in Princeton. In 1951, he shared the first Einstein award, and, in 1953, Princeton appointed him as a professor. Later he contributed to Albert Einstein's [1879-1955] relativity theory (discovering an unexpected solution to the field equations of gravitation), philosophy, and Set Theory (establishing the consistency of the axiom of choice and of the generalized continuum hypothesis with other axioms of Set Theory). Gödel

formal axiomatic systems and the provability of consistency. I will analogize the ongoing discourse in economic science to a similar debate that has occurred in mathematics and logic and explain why the results attained by Gödel and Church might be relevant for social policy (sections 5 to 8). Section 9 will provide comments on the implications of my hypothesis and section 10 will conlcude.

PART ONE

FAIRNESS V. WELFARE AS A CALL FOR CONSISTENCY IN SOCIAL DECISIONMAKING

2. The claim of *Fairness v. Welfare*

FVW makes the claim that "social decisions should be based *exclusively* on their effect on the well-being of individuals – and, accordingly, should not depend on notions of fairness, justice, or cognate concepts."[5] It intervenes in a long-lasting dialogue on what values should guide social decisionmaking that has involved lawyers, economists, and philosophers.[6] The primary question is whether welfare economics can be employed as a tool for social decisionmaking. A related issue is whether it should still be employed when its policy recommendations conflict with our sentiments of justice, fairness, equality, and other moral values.

Confronted with the question of whether we should take those decisions that we feel just or those that appear to maximize individuals' well-being, FVW openly stands for the latter and goes well beyond by contending that instances of fairness and justice should be given no consideration when they conflict with the maximization of individuals' well-being. If they are in accordance with the maximization of individuals' well-being, any independent account of such values is redundant and only serves as a proxy[7] for the welfarist criterion.

3. Dissecting the logic of *Fairness v. Welfare*

In this section, I will endeavor to unveil the logical framework that sustains the argumentation employed in FVW. In order to render the steps I take intelligible, I will use the following sentential

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died in 1978 in New Jersey. See Dawson (1988).

[5] FVW, p. xvii, emphasis in the original text.

[6] For an account of the discussion, see FVW throughout the book in relation to the various topics. See also Johnsen (1986) framing the origin of the discourse.

symbols and logical connectives:[8]

W	=	(the implementation of) the welfarist criterion, a set of criteria based on welfare economics;
F	=	(the implementation of) notions of fairness, a set of criteria based on fairness, justice, or related concepts;
maxW	=	the maximization of individuals' well-being;
d	=	the outcome is desirable;
$\neg$	=	'not', it negates the statement that follows;
$\rightarrow$	=	'if ... then', it connects two statements, the former of which implies the latter; the former is a sufficient condition for the latter, and the latter is a necessary condition for the former;[9]
$\leftrightarrow$	=	'if and only if', it connects two statements that imply one another; they are necessary and sufficient conditions for one another.

After clarifying the definitions of **W** and **F**, I will bisect the logic of the argumentation of FVW into a normative claim – concerning the aim of social decisionmaking – and a logical claim – concerning the evaluation of these two opposed criteria of choice in relation to such an aim.

The logical validity of the arguments made in FVW should be kept carefully separated from their soundness. Validity derives from the fact that the reasoning is conducted following correct rules of inference, according to which *if* the premises are true *then* the conclusion is also true. Whether or not the premises are true is what makes an argument sound or unsound, respectively. This raises a different issue that does not affect the validity of the argument. In the next two sections, I will first show that the arguments made in FVW are valid under simple sentential logic and then discuss their soundness.

3.1. Welfare and Fairness

FVW precisely defines the welfarist criterion **W** and the concept of notions of fairness **F** as

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[7] See footnote 2.

[8] Different symbols are often used to label the same logical connectives. I use the convention adopted in Enderton (2001, p. 14). Moreover, atomic statements are usually indicated by single upper-case letters; however, I also use lower-case letters (as **d**) and acronyms (as **maxW**) in order to render the reader's task easier.

[9] The statement **A**$\rightarrow$**B** is to be read as '**A** is a sufficient condition for **B**' (if **A** then **B**) or equivalently '**B** is a necessary condition for **A**' (**A** only if **B**).

mutually exclusive notions.[10] The welfarist method consists of utilizing the set of criteria provided by welfare economics as a way of maximizing welfare. In turn, the concept of welfare is defined expansively, as including anything that is of concern for individuals and excluding anything that is irrelevant to them (FVW, pp. 15-38 and 409). Thus, this notion of welfare also incorporates sentiments of fairness, justice, equality, and the like. **W** is, as opposed to notions of fairness **F**, intended as comprising any value like fairness and justice when they are taken into account irrespective of the effects they have on welfare (FVW, pp. 38-50). Hence, **F** includes any ideal that cannot be reduced to the welfarist criterion (FVW, p. 39).

Note that, in the set of possible criteria of choice for social decisionmaking, a criterion is either contained in **W** or it is not, and in the case of the latter, the criterion is contained in **F**.[11] Therefore, **F** may also be written as ($\neg$ **W**). Put differently, **W** and **F** are two mutually exclusive subsets of the set of all possible criteria, and every criterion in the set falls either in **W** or in **F**.

3.2. The normative claim

As a part of its normative component, FVW takes a consequentialist approach. That is, it does not evaluate the criteria of choice **W** and **F** per se, but considers them in relation to the outcomes they produce, and namely, in relation to the level of individuals' well-being they yield. Correspondingly, FVW claims that the maximization of individuals' well-being ought to be the aim of social decisionmaking. It does so in two steps.

First, it claims that advancing individuals' well-being is a desirable outcome while recognizing that such a claim is based on a value judgment on which general consensus may only be supposed.[12] FVW claims that *if* individuals' well-being is maximized, *then* the outcome will plausibly be seen as desirable by most people. In symbols:

(1) $\mathbf{maxW} \rightarrow \mathbf{d}$.

Second, FVW claims that reducing individuals' well-being (in order to advance notions of fairness) yields an outcome that is not desirable. This step is supported by 'inductive generalization' as it

[10] See also Kaplow and Shavell (2001, p. 283), for a formal definition, and (2003, pp. 332-335) for a focused summary of the arguments made in FVW.

[11] If we let **C** denote the set of all criteria for social decisionmaking, we have that $\mathbf{W} \cup \mathbf{F} = \mathbf{C}$ and $\mathbf{W} \cap \mathbf{F} = \{\emptyset\}$. That is, any criterion for social decisionmaking always falls in either subset and never in both.

[12] "[A] value judgment is involved in positing that individuals' well-being should be our sole concern. We suppose, however, that most of us do believe that individuals' well-being matters, and we suspect that the fact that any notion of fairness may involve making everyone worse off will be seen as troubling." FVW, pp. 468-469.

departs from an analysis of paradigmatic cases in which fairness is advanced at the expense of welfare and inductively implies that the conclusion may be generalized to all cases. In addition, this step involves the same value judgment as the former one, and it cannot be rigorously proved. Formally, the claim is that *if* individuals' well-being is not maximized (is reduced), *then* the outcome is not desirable. In symbols:

(2) $(\neg\, \mathbf{maxW}) \rightarrow (\neg\, \mathbf{d})$ or $\mathbf{d} \rightarrow \mathbf{maxW}$.

The compound statement on the right-hand side is equivalent to the one on the left-hand side; it is simply written in a different way.[13] To see why this must be the case, consider the following reasoning. If reducing individuals' well-being is a non-desirable outcome (as stated by the left-hand side), then the only way for an outcome to be desirable is if the individuals' well-being is maximized (as stated by the right-hand side, which says: *if* the outcome appears to be desirable, *then* individuals' well-being is maximized; in other words, the outcome appears to be desirable *only if* individuals' well-being is maximized).

Thus, the two premises of FVW yield that **maxW** implies **d** and, conversely, **d** implies **maxW**. Since **d** and **maxW** imply one another, we can write as a conclusion that the outcome is desirable *if and only if* individuals' well-being is maximized:

(3) $\mathbf{maxW} \leftrightarrow \mathbf{d}$.

Given that the two premises of the normative argument are based on a value judgment, the soundness of the resulting conclusion in (3) also rests on the same ground. As such, since the premises cannot be proved or disproved (they are value judgments), the conclusion, though logically valid, cannot be proved or disproved. More precisely, the conclusion is false only if the value judgment that supports the premises is rejected. Thus, it is neither false nor true; it is, rather, an opinion with which one might agree or disagree, as the authors clearly explain. In FVW, this argument provides the normative support to the endorsement of criteria that advance individuals' well-being and the exclusion of any criteria that reduces it. The next section will analyze this issue.

3.3. The logical claim

Once the goal of social decisionmaking has been established on the basis of a value judgment, the

[13] The logical law applied to (2) is known as contraposition. See Enderton (2001, p. 27).

problem arises of singling out criteria that achieve that goal. FVW considers two candidates: welfare-based criteria **W** and notions of fairness **F**. FVW proves that **W** and only **W** achieves the proposed goal. The structure of the argument is analogous to the former reasoning, but instead of deriving soundness from a value judgment, its truth is based on the way that the concepts in the premises are defined. Again, we break the argument into two steps.

First, FVW claims that the implementation of the welfarist criterion yields to individuals' well-being being maximized. That is, *if* the welfarist criterion is applied, *then* individuals' well-being is maximized.

(4) $\mathbf{W} \rightarrow \mathbf{maxW}$.

Second, FVW claims that employing notions of fairness **F** does not result in welfare maximization: *if* notions of fairness are employed, *then* individuals' well-being is not maximized:

(5) $\mathbf{F} \rightarrow (\neg\, \mathbf{maxW})$.

As we have already noticed, since **F** is defined as comprising any value that is different from the welfarist criterion, **F** may also be written as ($\neg$ **W**). Thus, $\mathbf{F} \rightarrow (\neg\, \mathbf{maxW})$ may be written as:

(6) $(\neg\, \mathbf{W}) \rightarrow (\neg\, \mathbf{maxW})$ or $\mathbf{maxW} \rightarrow \mathbf{W}$.

The right-hand side is simply the left-hand side rewritten differently.[14] The right-hand side states that *if* individuals' well-being is maximized, *then* the welfarist criterion is applied.[15] In other words, individuals' well-being is maximized *only if* the welfarist criterion is applied. Since we learn from premises (4) and (6) that **W** and **maxW** reciprocally imply one another, we can validly conclude that individuals' well-being is maximized *if and only if* the welfarist criterion is applied.

(7) $\mathbf{W} \leftrightarrow \mathbf{maxW}$.

The argument is valid; let us consider its soundness. Premise (4) must be true as it tautologically

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[14] See footnote 13.

[15] As we noticed before, the only concern here is to examine the logical structure of the arguments. The order in which statements are presented should not induce the reader to think that the causal relationship between them goes in the same direction. Clearly, (6) states that the maximization of individuals' welfare *implies* the implementation of welfare economics and not that the maximization of individuals' welfare *causes* the implementation of welfare economics. Causation runs in fact in the opposite direction.

states that if we (correctly) employ a criterion that, by hypothesis, is supposed to yield the maximization of individuals' well-being, then welfare maximization must result. Given that welfare economics is defined precisely this way, premise (4) must be true.

Arguing that premise (6) is true is the main focus of FVW; most of the authors' effort and of the book's length is devoted to showing that, in fact, if a criterion different from **W** is employed, then welfare will (or might) decrease.[16] However, also in this case, the premise derives its soundness from the definition of fairness **F** as ($\neg$ **W**), that is, as including any such notion that does not give explicit and exclusive account to individuals' well-being. "That advancing notions of fairness reduces individuals' well-being is in fact tautological on a general level. By definition, welfare economic analysis is concerned with individuals' well-being, whereas fairness-based analysis (to the extent that it differs from welfare economic analysis) is concerned with adherence with certain stipulated principles that do not depend on individuals' well-being. Thus, promoting notions of fairness may well involve a reduction of individuals' well-being." (FVW, p. 7).[17] In addition to that, FVW provides several paradigmatic examples in which it is shown that notions of fairness do reduce individuals' well-being.

As for the normative claim, it seems that the premises can be neither proved nor disproved as they follow tautologically from the given definitions of the concepts under analysis. The same can be said for the conclusion. The problem thus shifts to the definitions, which, in turn, are a choice of the analyst. To conclude, by bringing together the normative claim and the logical claim and combining statements (3) and (7), it results that the welfarist criterion, and only the welfarist criterion, leads to a desirable outcome and thus ought to be embraced.

(8) $\mathbf{W} \leftrightarrow \mathbf{d}$.

I stated in the introduction that these claims cannot be wrong. We are now able to appreciate that they are valid and that their soundness necessarily derives from the value judgment and the tautological definitions that ground the subsequent analysis.

[16] This argument was first advanced in Kaplow and Shavell (1999) and then formalized in Kaplow and Shavell (2001).

[17] FVW, p. 58: "It is true that it is virtually a tautology to assert that fairness-based evaluation entails some sort of reduction of individuals' well-being, for notions of fairness are principles of evaluation that give weight to factors unrelated to individuals' well-being". See also Kaplow and Shavell (2003, p. 335). The tautological nature of the problem had already been emphasized by Johnsen (1986, p. 269).

4. Interpreting *Fairness v. Welfare* as a call for consistency

On examination, the claim of FVW rests on premises that are taken as axioms (and hence assumed true) or are tautologically true. If one agrees with them, one cannot disagree with the conclusion. However, apart from a number of convincing examples, FVW does not attempt to prove that those starting points should be necessarily and generally accepted. In fact, FVW does not prove that the maximization of individuals' well-being *ought* to be the only criterion of policy assessment. Rather, FVW proves that *if* the maximization of individuals' well-being is taken as the aim of social decisionmaking *and if* welfare economics and notions of fairness are defined as explained, *then* the consequences that follow from the implementation of notions of fairness are inconsistent with (contradict) the outcomes of the welfarist criterion.[18] Put differently, if both **W** and **F** are employed, then policy decisions taken according to **W** will be inconsistent with those taken according to **F**. I will therefore look at FVW from this perspective and argue that its kernel consists of a call for consistency.

FVW may in fact be interpreted as stating that the criteria employed to evaluate social policy must be consistent with each other. The numerous examples made in FVW can be understood consequently: choices made according to welfare economics conflict with those made under notions of fairness. Thus, in order to advance a coherent social policy, it is necessary to employ only those criteria that are consistent with one another; that is, either **W** or **F** should be employed but not both. In fact, a system only based on **W** would yield consistent policy decisions. Likewise, a system only based on **F** would also yield consistent policy decisions. The choice between **W** and **F** remains, even in FVW, an issue of value judgments.

Two important problems remain to be addressed in the rest of this article. First, building on FVW, a consistent set of criteria is a necessary condition for consistent decisionmaking, but is it also sufficient? That is, given a consistent set of criteria of choice (either **W** or **F**), will the resulting social decisions always be consistent with each other? Put differently, are we able to prove the consistency of FVW as a whole? In addition, is a consistent set of criteria sufficient to resolve *all* social issues? Put differently, is FVW complete? The next several sections will show that these two questions are not trivial and, indeed, they pose some important problems to the analyst.

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[18] See on this point Kaplow and Shavell (1999, p. 76), "as a matter of logical consistency, a person who embraces a notion of fairness must in some occasions favor adopting legal rules that would make every person worse off. Of course, reason alone does not tell us that it is wrong to choose legal rules that hurt everyone."

PART TWO

CONSISTENCY AND COMPLETENESS

5. The nature of economic thinking in *Fairness v. Welfare*

FVW endorses the following decisionmaking procedure for social policy:

a) Take the preferences of individuals as given;[19]

b) Take the maximization of individuals' well-being as the desired goal;

c) Use the rules of welfare economics to deduce the appropriate social policies.

As in formal axiomatic systems, FVW lays down a number of unproved starting points or axioms (individual preferences, the goal of maximizing individuals well-being, and other postulates of welfare economics) and certain rules of inference (the rules according to which maximization is attained under welfare economics) in order to produce an infinite amount of policy recommendations.

Moreover, in FVW, a great deal of effort is expended in cleansing the system from any external influence, as most of the book is meant to support the claim that axioms that are inconsistent with those explicitly supported by welfare economics should not be accepted. It is suggested that any policy recommendation should be produced internally by such system, without interference from external criteria.

A formal attempt to investigate the axiomatic foundations of FVW largely exceeds the scope of the present inquiry. Nevertheless, I will argue that FVW may be considered as an attempt to reduce social decisionmaking to a system that gives precise and unequivocal solutions to any issue. Certainly, FVW is a formal approach to policy making. In the following sections, I will show that it might be important to verify what kind of formal system FVW actually embodies. In order to do so, I will refer to some important results attained in logic and show by applying these results that the problem of consistency is inherently related to the problem of completeness.

[19] In FVW, pp. 413-431, it is acknowledged that preferences may not be inalterable and may change as a result of certain social decisions. It is argued, however, that the outcome of alternative policies can be evaluated according to the maximization of individuals' well-being even if individual preferences are a function of such policies.

6. Consistency and completeness of axiomatic systems

A formalized axiomatic system is characterized by extreme precision in stating propositions and proving them. It is based on a definite vocabulary (a list of symbols), rules of formation (rules on how symbols can be combined in order to form propositions), and rules of inference (rules on how a proposition can be derived from other propositions). As a starting point, certain propositions (the axioms) are accepted without proof.[20]

The system is consistent if it is not possible to prove a proposition and its negation. The system is complete if all true propositions can be proved in the system.

We have already seen that FVW tends to remove the possibility of inconsistencies from social decisionmaking. However, it does not prove itself to be necessarily consistent. The possibility of inconsistencies cannot be ruled out by simply observing that no inconsistency has yet to be found since antinomies can be hidden even in systems that are seemingly consistent. For example, antinomies may be constructed even within Set Theory.[21] Because of the discoveries of antinomies and paradoxes within fundamental branches of mathematics, mathematicians have come to be convinced that unless consistency is rigorously proved, one could not be sure that no antinomy lies within an axiomatic system. The same might apply to social decisionmaking. Noting that jointly employing **W** and **F** yields to inconsistencies does not guarantee that a system based only on **W** (or only of **F**) is necessarily free of inconsistencies.

[20] For example, David Hilbert [1862-1943], in his study of the axioms sustaining Euclidean geometry published in 1899, argued that the whole system is built on 21 unproven statements from which all the other claims of Euclidean geometry can be deduced. In fact, the only difference between a theorem and an axiom is that axioms are accepted as true, while theorems have to be proven true by deriving them from the initial axioms.

[21] An antinomy was constructed by Betrand Russell [1872-1970] in 1901, and it is know as Russell's paradox. Consider the set of all lawyers; such a set does not contain itself since a group of lawyers is not a lawyer itself. We define as normal any set that does not contain itself. Consider now the set of all thinkable things; a set like this is abnormal as it contains itself; in fact, the set of thinkable things is itself thinkable. Now consider the question whether the set of all normal sets is normal or abnormal. If it is normal, it should not contain itself, but since it is the set of all normal sets, it must contain itself. Thus, it cannot be normal and it must be abnormal. However, if it is abnormal, it should contain itself, but since it only contains normal sets it cannot contain itself. Thus, it cannot be abnormal and it must be normal. Thus, we have obtained a contradiction. A popularization of Russell's paradox concerns a village in which some people get their own mail while others receive mail from a postman. Thus, the postman delivers mail to all those and only to those who do not get their own mail. The question arises how the postman gets his mail. If he gets his own mail, then a contradiction arises since he only delivers mail to those who do not get their own mail; thus he cannot deliver mail to himself. If he does not get his own mail, a contradiction arises again since he delivers mail exactly to those who do not get their own mail; thus he should also deliver mail to himself. The paradox that arises from the popularization can be resolved in several ways. It can be said, for example, that the postman is not an inhabitant of the village and hence he is not subject to the mailing rules or that he does not receive mail or that no postman exists. Similar solutions are not readily available for the original version of Russell's paradox. Since traditional logical theory was unable to resolve the problem, the paradox provided a stimulant towards a deeper analysis of Set Theory and ways to rid it of contradictions. See Delong (1970, pp. 81-82).

The problem of completeness also bears on the issues discussed in FVW. If a system for social decisionmaking is incomplete, some social issues remain unsolvable, or at least cannot be decided by using the criteria in which the system is grounded. It is evident that, being plausibly impossible or at least undesirable to give no solution to policy issues, a solution will have to be found by application of some different criteria that, by definition, will be a notion of fairness (as any criterion that does not fall into the welfarist approach is defined as a notion of fairness).

The impossibility of proving something is well known in logic and can arise due to different reasons; one of which is inadequateness in the set of axioms on which the system is based. For example, in Euclidean geometry, it is not possible to prove that through a point outside a given line only one parallel to the line can be drawn.[22] The solution adopted by Euclid was to take this fact as an axiom.[23]

The study of this problem made it clear that formal proves can be provided not only on whether a certain claim is true or false but also on whether a claim can or cannot be proven. This means that the provability of a claim within a certain system can be inquired into in a similar way as the truth of the claim.[24] This result will prove to be relevant for our purposes. Unfortunately, not all

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[22] In *The Elements*, written around the year 300 BCE, Euclid [325-265 BCE] derives all his theorems of geometry from five postulates or axioms, the last of which (although phrased in a different way from that given in the text) is the parallel postulate. The postulate never convinced, and generations of mathematicians unsuccessfully attempted to remove it by showing that it could be proved from the other four. In 1767 – some 2070 years after *The Elements* – Jean d'Alembert [1717-1783] wrote that the problem of the parallel lines was "the scandal of elementary geometry". Shortly after, it was indeed proven that it was impossible to prove the postulate from the other four. The discovery was attained through the work of Carl Friedrich Gauss [1777-1855], János Bolyai [1802-1860], Nikolai Ivanovich Lobachevsky [1792-1856] and Bernhard Riemann [1826-1866]. Bolyai's father, Farkas Bolyai [1775-1856] had himself tried to prove the postulate for 40 years and did his best to dissuade his son from the same endeavor, one that he considered a dangerous waste of time that "can deprive you of all your leisure, your health, your rest, and the whole happiness of your life". Once it had been proved that the axiom had to be accepted as such, it was also understood that, being independent from the other postulates, it could be replaced by alternative postulates stating that "through a point outside a given line no parallel to the line can be drawn" or that "through a point outside a given line more than one parallel to the line can be drawn". These constructions gave way to the study of alternative geometrical systems called non-Euclidean. The space described in these systems contradicts our sensorial experience. In spherical geometry – where no parallel can be drawn – planes bend like the surface of a sphere.

[23] The continuum hypothesis provides another example of a problem that cannot be resolved within the system in which it arises. The problem pertains to Cantor's theory of infinite numbers. Georg Cantor [1845-1918] discovered that, although both the real numbers (the set of all points on a line, numbers like 0, ... , 1, ... , 1.2, ... , 1.34908, ... , 2, ... , 3.6, ... , 10.467245, ...) and the natural numbers (0, 1, 2, 3, ...) are infinite, their sets have different magnitudes: while the natural numbers are infinite but countable, the real numbers are infinite and uncountable (they are more than can be counted using the infinite natural numbers 0, 1, 2, 3,...). Cantor's proof was based on a simple though ingenious and innovative diagonal method; Gödel's proof of section 8 may also be considered as such; see Enderton (2001, chapter 3) on this point. The further problem was to understand whether a subset of the real numbers had magnitude equal to either the real or the natural numbers or something in between. Cantor unsuccessfully attempted to solve the problem. Only later, as a result of the studies by Gödel in 1938, and Paul Cohen [1934] in 1963-1964, it became apparent that both hypotheses are consistent with the other axioms of Set Theory.

[24] In Euclidean geometry, proving the unprovability of the claim concerning the parallel means that this statement must be considered as an axiom and not as a theorem of the system.

cases of incompleteness can be filled by adding an axiom to the system. Indeed, as we will see, there are problems that cannot be remedied.[25] Some formal systems (the most complex ones) remain incomplete no matter how many new axioms we add to the system.

7. The Hilbert program

Notions of consistency and completeness gained utmost importance in logic and mathematics during the 19th century. The discovery of antinomies in different branches of logic and mathematics and the acceptance of the possibility to prove the unprovability of some propositions urged research into methods which ensured that axiomatic systems were free of both evils of internal contradictions and unprovable truths.

Great impetus was given by the German mathematician David Hilbert,[26] who set forth the need to put mathematics into a rigorous axiomatic form[27] and the need to construct a method that would enable us to prove, with certainty, the consistency of such a system within the system itself.[28] Euclidean geometry, for example, was believed to be consistent because its theorems are true of space. Non-Euclidean geometries (which are false of our daily experience) were considered consistent because theorems could be mapped into theorems of Euclidean geometry.[29] Hence, if no contradiction emerges in the latter, the former must also be free of antinomies. However, the observation that no contradiction has arisen yet is not sufficient to conclude that no contradiction exists; thus, the consistency of Euclidean geometry needs to be proved. Moreover, proving the consistency of a system by reference to another system only transfers the problem from one system

[25] In computer science, a problem that cannot be solved is that of writing a program that checks whether another program is going to complete the set of operations that have been assigned or whether it is going to run indefinitely. This problem is known as halting problem, and Alan Turing [1912-1954] demonstrated in 1937 that it cannot be solved. The kernel of the argument in Turing (1937) is much similar to Gödel's proof of section 8. The halting problem cannot be solved because it entails a loop, as the program that is supposed to tell us whether another program will stop might not stop in turn. In both cases, the system is turned onto itself and statements *about* the system are analyzed *within* the system.

[26] See footnote 20.

[27] Since Euclid (see footnote 22), geometry was given an axiomatic foundation but it remained – until modern times and with only some sporadic exceptions – the only branch of mathematics to be completely and rigorously axiomatized.

[28] The Hilbert Program captured the reaction of mathematicians and logicians to the appearance of contradictions in Set Theory. It was elaborated by Hilbert in a series of papers published between 1899 and 1939; some of which were written with Wilhelm Ackermann [1896-1962] and with Paul Bernays [1888-1977]. In his famous speech *The Problems of Mathematics*, delivered to the Second International Congress of Mathematicians in Paris, Hilbert said that, "every mathematician certainly shares [...] the conviction that every mathematical problem is necessarily capable of strict resolution" and identified 23 mathematical problems that were awaiting solution. Among them, he mentioned Goldbach's conjecture (see footnote 41) and the continuum hypothesis (see footnote 23).

[29] Eugenio Beltrami [1835-1900] argued that the 2-dimensional non-Euclidean models of Bolyai and Lobachevsky could be interpreted within traditional 3-dimensional Euclidean geometry, and thus, the problem of consistency could be mapped into the consistency of Euclidean geometry too. See also footnote 22.

to the other and does not provide any final solution. The only way to prove with absolute certainty that a system is free of contradictions is to construct a proof of such a claim within the system itself.[30]

Others argued that mathematics had to be intended as a chapter of logic[31], and momentum was reached around 1910 when the most fundamental branch of mathematics, namely Number Theory – the study of the properties of non-negative whole numbers[32] – was actually reduced to an axiomatic form in a systematic way in *Principia Mathematica*.[33] Thus, the problem of proving Number Theory – and other branches of mathematics and geometry – consistent became the same as proving the consistency of certain parts of logic.

8. Gödel's and Church's theorems

For a short time at least, *Principia Mathematica* seemed to have put hearts and minds at ease. It was in this environment however, that Kurt Gödel[34] demonstrated that this system, and consistent systems of similar complexity, (1) contain true propositions that cannot be proved within the system and (2) cannot produce a proof of their own consistency.[35] Gödel's results[36] actually gave a negative

[30] Hilbert was looking for finitistic proofs, that is, proofs of consistency that could be expressed in a finite number of steps and that were not based on the consistency of any other external system.

[31] This conjecture is known as the Boolean program and is named after George Boole [1815-1864], who put the foundations for modern logic and computer science. It is associated with the work of 19th-century mathematicians on the foundation of analysis, whose various notions were to be expressed in number-theoretical terms.

[32] Number Theory studies the properties of whole numbers equal to or greater than zero (0, 1, 2, 3, ...), also called natural numbers, cardinal numbers or non-negative integers. The same branch of mathematics can also be referred to as arithmetic. The properties of natural numbers are statements such as 'there exist no biggest prime', 'any even number can be written as a sum of two primes', 'all numbers can be written as the product of prime numbers', 'equations of a certain type have solutions that are natural numbers', and so on.

[33] Bertrand Russell (see footnote 21) and Alfred North Whitehead [1861-1947] published the three volumes of *Principia Mathematica* between 1910 and 1913 (Whitehead and Russell 1997).

[34] See footnote 4.

[35] Gödel (1931). A classical accessible exposition of Gödel's results is provided in Nagel and Newman (2001). Hofstadter (1999), while providing a parallelism between logic (Gödel), drawings (Escher) and music (Bach), leads the reader through the main concepts behind Gödel's discoveries, their proof and their applications. Smullyan (1988) introduces and explains Gödel's second theorem through a number of logical puzzles. See also Smullyan (1992). For a technical account, see Smullyan (1961, chapters 1 and 3) and Enderton (2001, chapter 3). A very illuminating semi-technical account of Gödel's results, Church's theorem, and various techniques to prove them is given by Barkley Rosser [1907-1989] in Rosser (1939). DeLong (1970, chapters 2.12 and 5) guides through the link that connects logical paradoxes, unproven mathematical truths and Gödel's and Church's theorems. It also discusses the philosophical implications of these results. On Gödel, see also Wang (1990 and 1997).

[36] More precisely, Gödel's first theorem states that "For suitable *L* [which stands for formal system or logic that is able to treat natural numbers and their addition and multiplication], there are undecidable propositions in *L*; that is, propositions **F** such that neither **F** nor $\neg$ **F** is provable". Gödel's second theorem states that "For suitable *L*, the simple consistency of *L* cannot be proved in *L*". See Rosser (1939, pp. 54-55).

answer to both problems of verifying completeness and proving consistency.[37] The complexity of the proof of Gödel's theorems is comparable only to the impact they had on the subsequent development of logic and computer science, but a general understanding of the way he proceeded is of interest for our purposes.

First, he found an ingenious system to write theorems of Number Theory in numbers.[38] That is, he managed to write the theorems *about* numbers in a language only based on the numbers themselves.[39] The enterprise is painstaking but quite mechanical. He assigned a number to all mathematical symbols used to write theorems and proofs. For example, the symbol '0' (for zero) was assigned the number 1, the symbol '¬' that we used in section 3 was assigned the number 5, and so forth. He then defined grammatical rules so that the numbers could be meaningfully combined in order to express formulas, theorems, and proofs.[40] The transcription process is time-consuming and involves a great deal of patience and space because theorems written as strings of numbers are much longer than the same theorems written with the symbols commonly used in mathematics. However, its logic is conceptually easy to grasp: written language consists of a series of symbols and rules on how to combine them. English uses letters and punctuation; the Gödel language uses numbers instead.

The importance of this translation resides in the fact that now theorems *about* numbers are written *in* numbers. In fact, once the theorems about numbers are written in numbers, a mathematical proof of a theorem – that is a sequence of steps by which the theorem can be derived from the axioms of the system – can be expressed as a series of properties of the numbers associated with the theorem. Hence, the system, in a way, can be made capable of reasoning about itself.

The next step toward a proof of incompleteness is to find a proposition and prove that it is true and can be expressed in the system but that it cannot be proved or disproved. If such a proposition

[37] Godel's title (1931), 'On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems I', reveals that he intended to publish a sequel, as he feared that the arguments were too closely related to *Principia Mathematica* and needed to be extended to a broader set of axiomatic systems in order to be fully understood. Instead, his results were quickly accepted in their broader import, and no second version of the paper was needed.

[38] This procedure is known as Gödel numbering or arithmetization of metamathematics, and it was also employed by Alfred Tarsky [1901-1983] in a paper published in 1933 but in preparation since 1929. For examples of the Gödel numbering, see Nagel and Newman (2001, pp. 68-80).

[39] Findlay (1942) explains how the logic of Gödel's proof can be carried out in ordinary language without the help of the Gödel numbering. It shows how a sentence can be constructed that says of itself that it cannot be demonstrated, hence arriving at the paradox explained in the text. It also helps in understanding the mechanics of the proof without the complexities of the original formulation.

[40] The translation process is based on a specific property of natural numbers: any number is either prime (it can be divided only by 1 and itself) or the product of unique prime numbers with a certain exponent. Using this property, it is possible to construct a numerical language capable of expressing formulas, theorems, and proofs of theorems in an

can be found, then it is demonstrated that the system is not able to decide (that is, prove or disprove) all problems that may arise within it: the system is incomplete, as there are certain truths that cannot be demonstrated.[41]

In this respect, an analogy may be helpful. Greek philosophers were already aware of a logical paradox. If one says, "I am lying," one's statement can be proved neither true nor false.[42] In fact, if the statement is proved true, then one is really lying and hence the statement must be false. Conversely, if the statement is proved false, then one is not lying and hence the statement must be true. In both cases, we end up with an undecidable problem. Gödel constructed something similar: he wrote a proposition **G** that simply asserted "I am a proposition that cannot be proved true." For reasons that are similar to the ones given for the liar paradox, the proposition **G** can be proved neither false nor true. In fact, if **G** could be proved true, then what it states must be true, and hence, there would be a contradiction. Likewise, if **G** could be proved false, then what it states turned out to be true, and there would again be a contradiction.

Nevertheless, the statement definitely belongs to Number Theory as it is written in numbers (recall that Gödel used a language only based on numbers), and it is in fact true since it precisely states that a certain theorem cannot be proved, which turns out to be true. The fact that **G** cannot be demonstrated is true but cannot be demonstrated.

The Gödel sentence **G** is an example of a true proposition that cannot be proved in Number Theory, and, thus, Number Theory is incomplete.[43] Gödel further noticed that the situation does not

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unequivocal manner.

[41] An example of a very simple statement that is true, though yet unproved in arithmetic, is know as Goldbach's conjecture. It states that every even number is the sum of two primes. No even number has been found that does not have this property (for example: 4=2+2, 34=13+11, and 84=17+67) but no formal proof is available yet. The conjecture was formulated by Christian Goldbach [1690-1764] in a letter he wrote to Leonhard Euler [1707-1783] in 1742. Goldbach also discovered that every odd number is the sum of three primes. A partial proof of this second conjecture was provided by Ivan Vinogradov [1891-1983] in 1937.

[42] This paradox has been known since the ancient Greeks as Epimenides paradox or liar paradox. Epimenides was a Cretan and said, "All Cretans are liars". More sharply, he might have said, "I am lying" or "This statement if false". Gödel (1931) refers both to the liar paradox, to another paradox known as the Richard antinomy – due to Jules Richard [1862-1956] – and to any other epistemological antinomy. He remarked that these arguments may be affected by a faulty circularity, which is instead absent in Gödel's proof (Gödel 1931, footnotes 14 and 15). Consider for example the liar paradox and assume that there are three Cretans and each of them is a liar. Let us label them as Liar1, Liar2, Liar3. If we assume that liars can only tell lies, it is obvious that nobody of the three can ever say that all Cretans are liars, because he would then tell the truth. Hence, the sentence "I am lying" or "All Cretans are liars" cannot simply be spoken by any of the three liars or the initial assumption that liars tell lies is violated. Thus, there is no paradox. Now assume that one of the three Cretans is a truth-teller. Then, only the liars can state that "All Cretans are liars", which is indeed a lie, as one Cretan is not a liar but a truth-teller. Thus, no paradox arises in this case either. The paradox fades when the initial specifications make some statements inadmissible. On the contrary, the Gödel theorem is not affected by this problem and cannot be dismissed on the basis of similar arguments.

[43] Gödel's results were later refined by Rosser (1936). The original formulation referred to ω-inconsistency, that is to the

change as we add axioms to the system because the problem reproduces itself at a higher level and never disappears. Hence, the Euclidean solution to add an axiom is not practicable in this case. He also noticed that the only way to make the system complete was to introduce some contradictions, hence making the system complete but inconsistent.

In addition, Gödel, building on the previous result (if a system is consistent then it is incomplete), proved that it is impossible to demonstrate the consistency of such a system within the system itself. He demonstrated that proving consistency implies the same kind of contradiction that we have seen arising while trying to prove the sentence **G**. From there, the paradox arises that we can prove that a system is consistent only if it is not.

Not long after the publication of Gödel's article, Alonzo Church[44] showed that not only do formal systems contain undecidable propositions but also that it is impossible to determine in an effective way which propositions are undecidable. That is, it is not possible to determine ex ante what are the problems that a certain formal system is able to solve and what will remain unsolvable.[45]

Gödel remarked in an addendum in 1963 that his results were not specific to the system of *Principia Mathematica* and to Number Theory, to which he directly referred, but rather they apply in a very general way. In fact, "in *every* consistent formal system that contains a certain amount of finitary Number Theory there exist undecidable arithmetic propositions and [...], moreover, the consistency of any such system cannot be proved in the system."[46] In fact, both of Gödel's results are valid for any formal axiomatic system that is powerful enough to express natural numbers and operations of addition and multiplication among them. Simpler systems are not affected by incompleteness and can prove their consistency.

Undoubtedly, Gödel's results puzzled logicians and mathematicians. Should they also puzzle

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following case: it is possible to prove a certain property of numbers in a general way and, at the same time, it is possible to prove that number 1 does not have the property, number 2 does not have the property, and so forth for all numbers. A certain property is proved in general and disproved for each of its occurrences. Simple inconsistency is a slightly different concept, as it implies that a property is proved and disproved in a general fashion at the same time. If a system is inconsistent, it is also ω-inconsistent, while the opposite is not necessarily true. Further developments were due to Alonzo Church [1903-1995], Stephen Kleene [1909-1994] and Alan Turing (see footnote 25). In particular, Kleene (1936) provides a third (after Gödel and Rosser) approach to prove Gödel's first theorem.

[44] See Church (1936) and footnote 43.

[45] More precisely, Church's theorem states that, "For suitable *L*, there exists no effective method to determine what propositions of *L* are provable." An effective method is one whose steps are precisely determined and that is certain to produce an answer in a finite number of steps. See Rosser (1939, pp. 55-56).

[46] Gödel (1931, note added on 28 August 1963), emphasis in the original text. In Gödel (1931, p. 173), it was already observed that the results are "not in any way due to the special nature of the systems that have been set up, but [hold] for a wide class of formal systems".

law and economics scholars?

9. Gödelean Implications for Policy Making

If transplanted into social policy, the results attained by Gödel and Church describe a world in which it is impossible to design a formal system for social decisionmaking that is at the same time consistent and complete. The two properties trade off each other.

I have interpreted FVW as an attempt to remove inconsistency from social decisionmaking by assuring that no incompatible criteria coexist within the system. FVW identifies welfare as a candidate for guiding policy assessments and contrasts the use of any other criteria that might be potentially conflicting with welfare. An issue not discussed in FVW is the completeness of a system exclusively based on welfare. Gödel's first theorem shows in fact that the struggle toward consistency might in fact compromise completeness.

A consistent FVW is a system for social decisionmaking in which no contradictory decisions can be taken by use of the criteria of choice endorsed in the system, but for FVW to be complete, all social issues must be addressable. Applying Gödel's result, it might be the case that some relevant social issues do not find a solution in a consistent FVW. Gödel's results suggest a way to overcome this problem: FVW must be inconsistent to be complete, which means employing simultaneously both welfare and some notions of fairness as criteria of choice, at least to some extent.

The coexistence of conflicting criteria is paradoxically responsible for an evil while remedying another. From this perspective, the claim that welfare and fairness should not be employed simultaneously must face the challenge that unsolvable problems pose. The question becomes whether society values more consistency rather than completeness or how much inconsistency should we accept in order to make up for incompleteness.

A relevant, related social problem is one of identifying and solving the issues that a consistent FVW will not be able to address. Church's theorem suggests that there exists no systematic procedure by which all such issues may be clearly identified ex ante. This means that unsolvable issues have to be identified and tackled when they arise in an individual fashion. In general, the analysis of the types of issues subject to Gödelean incompleteness in logic suggests that recursivity is the source of the problem. We have seen that the sentence **G** that Gödel proved to be unprovable is constructed in a way that makes the system turn onto itself. As in the liar paradox, the **G** proposition creates a sort of logical loop – a connection between a statement that is made *about* the system and a statement that is made *in* the system. The statement "I am lying" refers both to the content of the statement and to the statement itself. Recursivity is also the core of the halting problem in computer science.[47]

Economics is powerful enough to address questions that create a similar sort of recursivity. One problem that is discussed in FVW, for example, concerns the definition of the optimal policy in those situations in which individuals' preferences are endogenously dependent on the policies.[48] Thus, choosing a certain policy also implies choosing for a certain set of individual preferences on the basis of which the policy will have to be evaluated (as the welfarist criterion exclusively bases policy assessments on individual's well-being and the latter depends on individuals' preferences). This might generate a certain indeterminacy as policy **A** might be preferable over any other policy alternative according to the preference set $\mathbf{P_A}$ that the policy induces, while policy **B** might be preferable over any alternative according to $\mathbf{P_B}$. As a result, it might be impossible to decide between **A** and **B** on the sole basis of welfare.

This example is not exhaustive of all kinds of incompleteness that may arise in social decisionmaking, but it should give an indication of the type of problems that incompleteness might cause. The policy maker who only relies on one consistent set of criteria might face similar dilemmas.

10. Conclusion

In *Fairness versus Welfare*, it is maintained that welfare and fairness, if jointly employed as criteria for social decisionmaking, lead to inconsistent policy assessments. In this article, I contend that an inherent limitation might affect even a consistent system for social decisionmaking only based on the welfarist criterion: the system might be incomplete. That is, there might exist an indefinite number of relevant policy issues that the system is simply unable to decide.

The impossibility to decide on certain policy issues has been the object of renowned studies. Arrow (1951), Sen (1970) and Kaplow and Shavell (2001) derive impossibility results from the implementation of inherently conflicting criteria. Arrow (1951) shows that there exists no design of social decisionmaking that simultaneously obeys a list of five basic axiomatic requirements. In Sen (1970), the impossibility of a Paretian liberal is proven by showing that the choices taken under the

[47] See footnote 25.
[48] FVW, pp. 413-418.

Pareto principle are incompatible with the requirements of a minimal notion of liberalism. In Kaplow and Shavell (2001), policy assessments made under any non-welfarist method are shown to violate the Pareto principle.

In these studies, the crux is to emphasize an inconsistency among certain conflicting criteria of choice. The impossibility for consistent policy assessments or social decisions to obey certain criteria derives from the inconsistency among these criteria.

On the contrary, the point of my analysis is that even a consistent set of criteria – as for example the welfarist one – might yield undecidability. In this case, undecidability does not derive from inconsistency but from an inherent Gödelean limitation of formal systems to express and treat all relevant issues. I derive this conclusion in an analogical way from similar results attained in the field of logic in the beginning of the nineteenth century. These results were accomplished within a debate on the completeness and consistency of formal axiomatic systems known as the Hilbert program. In these studies, it is shown that sufficiently complex, formal axiomatic systems that are consistent are also inherently incomplete and that there exist no effective way to list the propositions that the system is unable to prove.

If this is also true for the welfarist method (or any other consistent system for social decisionmaking), there will be relevant policy issues that cannot be decided within the system. This suggests that the assumption of unrestricted domain,[49] usually taken as a condition for the evaluation of relevant alternatives, might not always be defensible, as the realm of decisions that can be taken according to some social welfare or social decision function might not include all decisions that need to be taken.

On the policy level, the impossibility to decide certain issues may pose serious practical problems to the functioning of society. In this case, the use of mixed and yet inconsistent criteria of choice may be inevitable.[50] FVW contends that consistent policy assessments cannot derive from a system that jointly employs welfare and fairness as criteria of choice. Thus, only one of them should be used. I argue that a joint use might at times be inevitable to cope with the inherent incompleteness of the system.

[49] See Arrow (1951), Sen (1970, p. 153).

[50] It is worth remarking that the problems discussed in this article do not descend from practical difficulties in implementing the recommendations of FVW, but rather from the theoretical impossibility of designing a complete and consistent system of a certain complexity.

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