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Rents, Dissipation, and Lost Treasures with N Parties*

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Abstract

The rent-seeking literature is unanimous on the fact that, in a rent-seeking context, the rent dissipation increases with the number of potential participants. In this paper we analyze the participants' choice to enter the game and their levels of efforts. We show that the usual claim – that the total dissipation approaches the entire value of the rent – applies only when participants are relatively weak. In the presence of strong competitors, the total dissipation actually decreases, since participation in the game is less frequent. We also consider the impact of competitors' exit option, distinguishing between "redistributive rent-seeking" and "productive rent-seeking" situations. In redistributive rent-seeking, no social loss results when all competitors exit the race. In productive rent-seeking, instead, lack of participation creates a social loss (the "lost treasure" effect), since valuable rents are left unexploited. We further show that in N-party rent-seeking contests, the lost-treasure effect perfectly counterbalances the reduction in rent dissipation due to competitors' exit. Hence, unlike redistributive rent-seeking, in productive rent-seeking the total social loss remains equal to the entire rent even when parties grow stronger, irrespective of their number.

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1 Introduction

When resources are not – or not yet – subject to clear ownership, private parties tend to expend effort in order to gain control over them. Settlers occupy land in newly discovered regions, producers strive for monopoly power, pharmaceutical enterprises rush for patents, and researchers compete for new scientific findings. The rent-seeking literature initiated by Tullock (1967), Krueger (1974), and Posner (1975) has long analyzed these types of problems, concluding that they may result in expenditures that exceed the socially optimal levels. Critics have reproached the proponents of rent-seeking models for bringing about too negative a view of reality. Proponents of alternative views argue that rent-seeking models neglect the fact that rent dissipation is often the by-product of valuable competition in socially beneficial activities, such as scientific or technological research.[1]

This debate led to the important distinction between forms of redistributive rent-seeking, aimed at the reallocation or appropriation of a rent, and productive rent-seeking, where the competitors' expenditures are instrumental to the discovery or creation of new resources.[2] In this study, we examine both redistributive and productive rent-seeking games among N identical parties and challenge a rather uncontested result in the literature, namely, that in a rent-seeking game the total rent dissipation increases with the number of contestants.

We show that rent dissipation increases with the number of potential participants – and ultimately approaches the entire value of the rent – only when participants are relatively weak (in a sense that will be specified in the following). In contrast, when parties are relatively strong, an increase in the number of contestants actually leads to a reduction in the total rent dissipation.

Our analysis shows that competitions involving weak players lead to larger dissipation because weak contestants always play the game. Strong competitors, instead, randomize their participation in the game and an increase in the number of players induces them to play increasingly less often. This crowding-out effect leads to a reduction in the rent dissipation.

It should be noted that when the probability of participation is lower than 1, it is possible that no party enters the game and thus that the rent will remain unexploited. This occurrence does not amount to a social loss in redistributive rent-seeking, but it does so when rent-seeking efforts are conducive to a socially productive outcome. In these cases, the unexploited rent amounts to a social loss (which we call the "lost treasure" effect) that ought to be added to the rent dissipation. This paper shows that the total social loss so calculated is constantly equal to the entire value of the rent, irrespective of the number of participants.

In the following we put the current study in the context of the existing literature. Section 2 provides the formal analysis; some of the proofs are in the appendix. Section 3 concludes discussing potential extensions of our results and implications of our findings for social policy and R&D activities.

[1] See for example the introduction to Barzel (1997).

[2] See Dari-Mattiacci and Parisi (forthcoming) discussing this point in a 2-party game.

1.1 Rent Dissipation and Lost Treasures in Tullock's Paradox

Tullock's (1967) seminal paper on how rational parties expend resources in the pursuit of rents provided the basis for the understanding of how the degree of rent dissipation varies with the value of the prize, the number of contestants and the allocation rules. The early extensions of Tullock's (1967) insight by Becker (1968), Krueger (1974), Posner (1975), Demsetz (1976), Bhagwati (1982), and others hypothesized a full-dissipation equilibrium, similar to that generated by competitive markets.[3] In a long-run equilibrium, rents would be competed away by the contestants and rent-seeking investments would thus yield the normal market rate of return.

In his seminal work on "Efficient Rent-seeking," Tullock (1980) developed the insight that the marginal return to rent-seeking expenditures influences the total expenditures on rent-seeking activity. Tullock's (1980) results shook the conventional wisdom in the literature, identifying conditions under which competitive rent-seeking could lead to under- or over-dissipation. Tullock's analysis showed that, when investments in rent-seeking exhibit increasing returns, aggregate expenditures could exceed the contested prize. This could lead to negative expected returns for the players, making it rational for players to exit the game. But, if no player entered the rent-seeking contest, the prize would remain unclaimed.[4] Hence, Tullock's well-known paradox.[5]

In a recent paper, two of us tackled Tullock's paradox identifying the possibility that the parties adopted mixed participation strategies (Dari-Mattiacci and Parisi, forthcoming). The analysis considered the case of two (identical) players facing varying returns to effort, and showed that Tullock's paradox originates from a conflict between the decision whether or not to play and the optimal strategy when playing. The analysis differs from previous studies[6] by giving players the opportunity to choose simultaneously and independently (1)

[3] Most notably, Posner's (1975) full dissipation hypothesis became popular in the empirical literature and also had a strong appeal in the theoretical one. For a survey of the literature see Buchanan, Tollison, and Tullock, (1980), Congleton and Tollison (1995), Lockard and Tullock (2000), Rowley, Tollison, and Tullock (1988), and Tollison (2003).

[4] See also Rowley (1991) on the importance of this problem for the development of the idea of rent-seeking.

[5] With rational expectations, parties would realize that the rent-seeking contest would generate negative expected returns, and would consequently choose to exit the contest, if given an opportunity to do so. Tullock (1980) points out the paradoxical result that if no one enters the contest, any one contestant that enters the race would win the prize, regardless of the effort level he chooses. Therefore there is an incentive to enter, destabilizing the hypothesized no-participation equilibrium. Tullock thus concluded that the existence of negative expected returns when all parties participate cannot be used to infer that the equilibrium level of participation will always be zero.

[6] Baye, Kovenock, and de Vries (1994) introduced mixed strategies with varying levels of effort, but they obtain an explicit solution only in the discrete case. Hillman and Samet (1987) analyzed a slightly different version of the rent-seeking paradox, in which the rent is entirely adjudicated to the highest bidder as in an all-pay auction. In Pérez-Castrillo and Verdier (1992), the randomization is attained over the number of players that enter the game. Other solutions to Tullock's paradox have been sought by transforming the game into a dynamic one and by introducing asymmetries between the players.

whether to play and (2) how much to invest in rent-seeking activities.

In this paper, we build on this framework of analysis in order to discuss the problems of participation and optimal effort when N parties compete for a rent and face varying returns to rent-seeking efforts. We study how the expected expenditures in rent-seeking and the expected value of the lost treasure vary with the number of contestants and the marginal return to rent-seeking efforts.

1.2 Rent Dissipation and Lost Treasures with Multiple Rent-Seekers and Varying Returns to Effort

When parties randomize their strategies, it could happen that more than one party decides to play. Although none of the parties would rationally spend more than the full rent, the total expenditures of all parties could exceed the rent – a result that confirms Tullock's claim that increasing returns to investment may induce over-dissipation. With mixed strategies, the opposite may also be true. With random participation, available value may remain unexploited when no player enters the rent-seeking contest. In these situations the analysis of the social cost of rent-seeking should account for the possibility that valuable rents may remain unexploited. We refer to this possible loss as the "lost-treasure effect."

When parties are endowed with a weakly productive technology of effort, either with decreasing or constant returns to effort, they always enter the game and never randomize their strategy. In contrast, when they have increasing returns to effort, the parties' choices depend both on the marginal return to the individual effort and on the number of potential contestants.

The relative strength of the parties is measured as a function of these two variables. In both redistributive and productive rent-seeking games, when parties are relatively weak (that is, when their marginal return to effort is low relative to the number of parties) they always participate in the contest and the dissipation increases with the number of participants, ultimately approaching the whole value of the rent. When parties are relatively strong (that is, when their marginal return to effort is high relative to the number of parties), it is never advantageous for parties to invest in total more than the rent. In fact, when their strength increases, they tend to invest more in the contest, but also tend to participate less often.

As a result, when parties are stronger, the dissipation increases steadily up to the point at which parties expend in total as much as the rent and it becomes convenient not always to participate. After this point, in redistributive rent-seeking activities the dissipation decreases, while in productive rent-seeking activities it remains equal to the rent. This is because, in productive rent-seeking the reduction in the parties expected expenditures is perfectly counterbalanced by the expected value of the rent, which remains unexploited when no party takes part in the game.

A similar analysis applies with respect to changes in the number of potential participants. An increase in the number of potential participants reduces the incentive for each party to enter the race. This has different welfare implications

in the two cases of redistributive and productive rent-seeking. In the case of redistributive rent-seeking, the total dissipation of socially valuable resources corresponds to the sum of the parties' investments in rent-seeking activities. An increase in the number of potential participants, by reducing the incentive for each party to enter the race, may lead to a beneficial reduction in the dissipation of rents (i.e., a lower rent-dissipation effect).

In the case of productive rent-seeking, the parties compete for rents that are associated with socially valuable activities and hence a social loss arises not only from the parties' rent-seeking expenditures (i.e., the rent-dissipation effect), but also from the lack of exploitation of available rent (i.e., the lost-treasure effect). Here, an increase in the number of potential participants produces two countervailing effects: a reduction in the rent-dissipation effect and an increase in the lost-treasure effect. We show that the rent-dissipation effect is negatively related to the number of contenders, while the lost-treasure effect is positively related to it. Interestingly, the sum of the rent-dissipation effect and the lost-treasure effect does not depend on the number of contenders and equals the full value of the rent.

2 Analysis

2.1 The basic model

We consider $N \geq 2$ identical, risk-neutral individuals who may participate in a simultaneous contest with a prize equal to 1.[7] Each individual indexed by i chooses between two actions: either to enter the game with an effort $X_i \in [0, 1]$ or not to enter the game at all. Each individual is allowed to randomize over his or her pure strategies – that is, he or she may enter the game with probability p_i and not enter the game with probability $1 - p_i$.

Parties act as to maximize their own individual expected payoffs from the game, conditional to the behavior of the others. In order to calculate the expected payoff of each individual, it is necessary to specify the sharing rule for the prize and the way in which each participant anticipates the moves of the others.

To begin with, if an individual enters the game, he or she is awarded a share of the prize equal to $\frac{X_i^r}{X_i^r + \sum_{j \neq i} X_j^r}$. This share depends on the individual's investment X_i, together with the number of actual opponents who have entered the game and their respective effort X_j. It can be interpreted either as a real sharing of the prize or as a probability of winning the entire prize. Since parties are risk-neutral, both interpretations are formally equivalent. It is easy to see that the exponent $r > 0$ (the same for all the individuals) represents, as it is usual in the literature, an index of the individuals' productivity of effort, whose

[7] The assumption that the prize is equal to 1 is a choice made for merely methodological convenience and it is equivalent to measuring the parties' investments X_i as a fraction of the value of the prize, rather than in absolute terms, as it is more common in the literature. In this way it is easier to evaluate the rent dissipation.

value determines the value of the individual's marginal return to effort.[8] Finally, if no party participates the prize is not awarded.

In addition, and as it is usual when individuals play Nash strategies, we use the fact that the strategy choice of each individual is associated to a reasonable belief concerning the other players' strategies. There is an obvious way to formalize these beliefs: since no player can influence the decision of the others but takes them as given, and since the problem is symmetric, it is reasonable for each player to expect all of the others to play the same strategy. That is, we can consider that the opponents to player i enter the game with the same probability $p_j = q$, and exert the same effort $X_j = Y$, for any $j \neq i$. Consequently, we can rewrite the share as $\frac{X_i^r}{X_i^r + jY^r}$, where j is the number of opponents who have effectively entered the game together with player i. As a result, for each individual i the number of opponents who may be expected to enter the game is a random variable n distributed according to a Binomial law with:

$$\begin{aligned} B(j; N-1, q) &\equiv \binom{N-1}{j} q^j (1-q)^{N-1-j} \\ &= \frac{(N-1)!}{j!(N-1-j)!} q^j (1-q)^{N-1-j} \\ &= \Pr(n = j) \end{aligned}$$

corresponding to the probability that the number of opponents n playing (Y, q) be equal to $0 \leq j \leq N-1$.[9]

Thus, the expected payoff of each individual i may be now written as follows:

$$U_i(X_i, p_i; Y, q) = p_i \sum_{j=0}^{N-1} \Pr(n = j) \left(\frac{X_i^r}{X_i^r + jY^r} - X_i \right) + (1 - p_i)\, 0 \qquad (1)$$

and the individual efficient mixed strategy in the rent seeking game is defined as the solution to the maximization of the expected payoff in Exp. (1) over p_i and X_i. We have the following first order conditions:[10]

[8] More specifically, X_i^r is the total productivity or return to each individual's effort X_i. It is easy to see that, if $r > 1$, we have increasing marginal return to effort; if $r < 1$, we have decreasing marginal return to effort; if $r = 1$, effort has a constant marginal return. This interpretation was initially proposed by Tullock (1980).

[9] To illustrate, suppose that there are 5 parties. From the perspective of party 1, the probability that only parties 2, 3, and 4 participate and party 5 does not is given by $q^3(1-q)^1$. The term $\frac{(N-1)!}{j!(N-1-j)!}$ depicts instead the number of possible combinations of j participants in the pool of $N-1$ potential opponents. Going back to the example, the are $\frac{(5-1)!}{3!(5-1-3)!} = \frac{4!}{3!1!} = 4$ possible combinations of 3 opponents. In fact, we could have the following combinations of 3 opponents to party 1: (2, 3, 4), (2, 3, 5), (2, 4, 5), (3, 4, 5). For all of these combinations the probability of participation is always $q^3(1-q)^1$; thus, we need to take this probability into account 4 times. This exercise needs then to be repeated for any number j of potential opponents from 0 to $N-1$.

[10] That the solution is internal follows from the proof of proposition 1 given in the appendix.

$$\frac{\partial U_i}{\partial p_i}(X_i, p_i; Y, q) = \sum_{j=0}^{N-1} \Pr(n = j) \left(\frac{X_i^r}{X_i^r + jY^r} \right) - X_i = 0 \tag{2}$$

and

$$\frac{\partial U_i}{\partial X_i}(X_i, p_i; Y, q) = p_i \left[\sum_{j=0}^{N-1} \Pr(n = j) \left(\frac{jrX_i^{r-1}Y^r}{(X_i^r + jY^r)^2} \right) - 1 \right] = 0 \tag{3}$$

Exp. (2) yields that the expected payoff from participating in the game must be equal to 0, as it is the expected payoff of not entering the game. This is a standard condition for mixed strategies, stating that the opponents enter the game with such a probability $0 < q < 1$ and an effort $Y > 0$ that make party i indifferent between entering and not entering the game.

Exp. (3) simply states that the marginal increase in the expected share in the prize must equal the marginal cost of effort. This is also a usual condition, implying that the individually efficient level of effort when participating is such that a player weighs an increase in his expected return to effort against an increase in his cost of effort.

2.2 Equilibrium

Let us focus on the Nash equilibrium of this rent seeking game. With perfectly identical players, it is natural to consider the case of a *symmetrical* Nash equilibrium, such that $Y = X_i \equiv X$, and $q = p_i \equiv p$. In this case, Exp. (2) and (3) respectively become:

$$X = \sum_{j=0}^{N-1} \Pr(n = j) \left(\frac{1}{1+j} \right) \tag{4}$$

$$X = r \sum_{j=0}^{N-1} \Pr(n = j) \left[\frac{j}{(1+j)^2} \right] \tag{5}$$

We will now study the characteristics and the conditions of existence of this Nash equilibrium. The main results are summarized in the following proposition:

Proposition 1 *i) If $r > \frac{N}{N-1}$, the unique symmetrical Nash equilibrium is such that the N potential participants enter with a strictly positive probability p less than 1 and a strictly positive effort X, which solve (4) and (5). ii) If $r \leq \frac{N}{N-1}$, the unique symmetrical Nash equilibrium is such that the N parties enter with a probability equal to 1 and exert effort $X = \frac{N-1}{N^2} r$.*

The formal proof is given in the appendix. Proposition 1 yields that the individuals' choice between pure and mixed strategies only depends on the value

of the index r (describing the marginal productivity of effort) relative to the number of parties N. Defining as the strength factor of a competitor the term $r - \frac{N}{N-1}$, we will call 'strong' players those with a positive strength factor ($r > \frac{N}{N-1}$, requiring for example high returns to effort and/or many competitors), and 'weak' players those with a negative (or zero) strength factor ($r \leq \frac{N}{N-1}$, for example low returns to effort and/or few competitors).

In proposition 1, it is shown that for weak competitors $\left(r \leq \frac{N}{N-1}\right)$ the natural way to play the game is to adopt pure strategies, that is, it is optimal for all of the parties always to enter the game. On the contrary, for strong competitors $\left(r > \frac{N}{N-1}\right)$, it is rational to play mixed strategies and enter the game with a probability lower than 1.[11] Thus, loosely speaking, proposition 1 establishes that the rent-seeking contest is rendered less appealing for each contestant by an increase in the number of potential competitors (which implies a smaller share or probability to succeed) and/or by an increase in the players' return to effort (larger equilibrium expenditures). While relatively weak competitors are always ready to enter the contest, relatively strong ones prefer to confine their participation rate.

Figure 1 shows that the threshold level of $\hat{r} = \frac{N}{N-1}$ decreases when the number of parties increases, and asymptotically approaches 1 as N tends towards infinity.

INSERT FIGURE 1

This means that parties ought to be considered strong competitors at lower levels of r as N increases. In the next two paragraphs we separately discuss the case of weak competitors and the case of strong competitors and investigate the properties of the equilibrium behavior of the parties through a comparative statics analysis.

2.3 Weak parties play pure strategies

We have seen that when parties are weak competitors $\left(r \leq \frac{N}{N-1}\right)$ they find it always convenient to participate in the game, $p = 1$, with a positive effort level, $X = \frac{N-1}{N^2} r$,[12] which is increasing in the return to effort r $\left(\frac{\partial X}{\partial r} = \frac{N-1}{N^2} > 0\right)$, up to the point where the threshold $\hat{r} = \frac{N}{N-1}$ is reached, for which the effort attains the level $\hat{X} = \frac{1}{N}$. On the other hand, the effort is decreasing in the number of parties N (since $\frac{N-1}{N^2}$ is decreasing in N). In pure strategy, the payoff of each participant is equal to:

[11] It is easy to see that in this case, we also have $r > \frac{N}{N-1} > 1$, that is, strong competitors necessarily have increasing marginal returns to effort.

[12] The level of effort of an individual playing pure strategies maximizes the payoff $U_i(X_i, Y) = \frac{X_i^r}{X_i^r + (N-1)Y^r} - X_i$, which is given by substituting $p = q = 1$ in (1), since everyone participates in the game in equilibrium. This result is consistent with the previous literature. See Tullock (1980, p. 146).

$$U(X,1;X,1) = \frac{1}{N}\left[1 - r\left(\frac{N-1}{N}\right)\right]$$

which is non negative when $r \leq \frac{N}{N-1}$. It can be easily shown that the individual payoff U decreases both in r and N.

2.4 Strong parties play mixed strategies

If parties are strong competitors $\left(r > \frac{N}{N-1}\right)$, playing pure strategies would yield a payoff lower than 0 for each party. Tullock's paradox arises precisely from this occurrence. As shown in Dari-Mattiacci and Parisi (forthcoming) for the case of two symmetric parties, it is thus optimal for parties to randomize over their strategies and enter the game with a probability that is positive but lower than 1. The same result arises also in the case of N players: each party enters the game with a probability that makes the other parties indifferent between playing and not playing, which implies that the equilibrium expected payoff for each participant is equal to 0, as it is the payoff from not entering the game.

The study of the comparative statics[13] shows that when the return to effort increases, the equilibrium value of the probability of participation in the game decreases $\left(\frac{\partial p}{\partial r} < 0\right)$, while the equilibrium level of effort increases $\left(\frac{\partial X}{\partial r} > 0\right)$.[14] The intuition behind this result can be easily explained as follows. A higher return to effort induces each party to exert a higher level of effort in order to retain a larger share of the prize. However, when every party invests more in the game, their equilibrium shares in the prize remain constant, since the prize is equally shared among parties who make equal investments. Thus, they all bear a higher cost of effort which is not compensated by an increase in their share and hence results in a decrease in their payoff.[15] As a result, since the higher the productivity of effort, the higher the risk of receiving a negative payoff, parties tend to compensate an increase in the number of parties by reducing their probability of participation.

On the contrary, when the number of players increases,[16] both the probability of playing and the effort level of each party decrease. This is due to the fact that an increase in the number of parties makes the game less attractive, as each party faces the possibility to have to share the prize with a larger number of opponents. Thus, the greater the number of parties, the greater the individual risk associated with participation in the contest. To compensate, all parties reduce both their probability of entering, and their levels of efforts.

[13] See the appendix.

[14] This implies, as it is worth noticing, that playing mixed strategies gives an incentive to choose a *higher* effort than when playing pure strategies.

[15] Notice that the equilibrium value of the share in the prize is $\frac{X^r}{X^r + jX^r} = \frac{1}{1+j}$, where j is the number of actual participants in the game. Hence, in equilibrium, when each participant exerts the same level of effort, the prize is shared equally among them. But, the payoff in the event of j parties entering the game is $\frac{1}{1+j} - X$, and clearly decreases if X increases.

[16] Recall that N is a discrete variable.

INSERT FIGURE 2

3 The social loss of rent dissipation and lost treasure

In this Section, we consider the two cases of rent-seeking activities which have been exemplified in the literature: namely, redistributive and productive rent-seeking. The social losses associated to each of them are quite different. Redistributive rent-seeking is aimed at the reallocation or appropriation of a rent. Therefore, the social loss simply amounts to the total dissipation D, that is, the aggregate value of the resources invested by the parties.

In productive rent-seeking, instead, the competitors' expenditures are instrumental to the discovery or creation of new resources. For simplicity, we assume that the social value of the treasure is the same as its private value for the parties, which we have normalized to 1. In these cases, there is a second source of social loss that should be added to the rent dissipation D. When parties play mixed strategies, there is a positive probability that no party participates in the game, and thus, that the treasure will remain undiscovered. This lost treasure T is to be added to D in the calculus of the social loss, which becomes equal to $D + T$.

The actual measure of the social loss in the two cases depends on whether parties play pure strategies (the case of relatively weak contestants) or mixed strategies (the case of relatively strong contestants), and thus depends on the return to investment in effort, r, and on the number of parties, N.

3.1 The social loss with weak parties

When parties are relatively weak $\left(r \leq \frac{N}{N-1}\right)$ expected returns from rent-seeking are positive. Thus, parties always take part in the game. The total amount of resources dissipated in a rent-seeking activity is thus equal to the sum of the parties' efforts. Since parties always play, the treasure will always be found and hence the social loss of productive rent-seeking is the same as the social loss of rent seeking. Therefore, recalling that the individual level of effort is $X = \frac{N-1}{N^2} r$, we can write the social loss of redistributive and productive rent-seeking as a function of r and N, as follows:

$$D(r, N) = NX = \frac{N-1}{N} r \tag{6}$$

It is easy to see that D is increasing in r $\left(\frac{\partial D}{\partial r}(r, N) = \frac{N-1}{N} > 0\right)$, and increasing in N (for $N > 2$, the term $\frac{N-1}{N}$ is bounded from above by 1 and increases with N); moreover, it entails full dissipation for $r = \frac{N}{N-1}$. It is remarkable that, although the individual levels of efforts drop when the number of parties increases, the social loss still increases as an effect of more parties participating in the game, but it never overcomes the value of the rent.

3.2 The social loss with strong parties

When the rent-seeking contest involves relatively strong parties $\left(r > \frac{N}{N-1}\right)$, the risk of over-dissipation induces them to participate in the game with a probability lower than 1. Therefore, since the total number of participants is described by a random variable with a binomial distribution, the ex ante value of the rent dissipation due to the parties' efforts is given by the mean value of the number of participants (which is equal to Np) times the individual level of effort (X): $D(r, N) = NpX$.[17] In order to analyze the impact of changes in r or N, let us rewrite the rent dissipation as follows:[18]

$$D(r, N) = 1 - (1 - p)^N$$

In redistributive rent-seeking activities, the social loss is only equal to the dissipation. In productive rent-seeking activities, instead, since parties only play with a probability lower than 1, it is possible that treasures will not be found, as it may happen that no party enters the game. This "lost treasure" loss may be written as:

$$T(r, N) = (1 - p)^N$$

In other words, the value of the dissipation is equal to the total probability of participation, while the lost treasure is equal to the probability that the game is not played. Thus, it is easy to see that the sum of the rent dissipation and the lost treasure is always equal to the value of the prize, whatever the level of return to effort r or the number of parties N:

$$D(r, N) + T(r, N) = 1$$

From this result it is easy to calculate how rent dissipation and lost treasure vary when the parties' return to effort and the number of competitors increase.[19]

Regarding variations in the return to effort, we have already seen that parties tend to play less often when their return to effort increases. Thus, it is obvious

[17] As for the previous case where weak parties do not randomize, the ex ante dissipation is still the (expected) value of the total rent dissipation including the random participation of the N parties; thus, by definition we have: $D(r, N) = \sum_{j=1}^{N} \Pr(n = j) jX$, where $\sum_{j=1}^{N} \Pr(n = j) j = Np$.

[18] Using (4), we have:
$D(r, N) = Np \sum_{j=0}^{N} \binom{N-1}{j} p^j (1-p)^{N-j} \left(\frac{1}{1+j}\right)$
$= \sum_{j=0}^{N-1} \frac{N(N-1)!}{(j+1)j!(N-(j+1))!} p^{j+1} (1-p)^{N-1-j}$
$= \sum_{j=1}^{N} \binom{N}{j} p^j (1-p)^{N-j}$
$= 1 - (1-p)^N$

[19] It is worth noticing that all the effects thereafter are driven by the fact that the equilibrium values of both dissipation and lost treasure depend only on the equilibrium value of the probability with which the parties enter the game - and not on their levels of effort.

that the lost treasure will increase as a result: $\frac{\partial T}{\partial r}(r, N) > 0$. Consequently the value of the dissipation must decrease: $\frac{\partial D}{\partial r}(r, N) < 0$.

Concerning the impact of variations in the number of players, we have seen that the probability of participation decreases when the number of parties increases, even if there are more parties who could eventually participate. Therefore, also in this case, the lost treasure increases, and thus symmetrically the dissipation decreases.

INSERT FIGURE 3

4 Conclusions

In this paper, we consider an important aspect of Tullock's (1980) rent-seeking paradox, generating results that run contrary to an established consensus in the rent-seeking literature. We show an interesting relationship between number of contestants, returns to rent-seeking investments, and total rent dissipation when parties have an exit option and are allowed to undertake mixed participation strategies.

Rent dissipation increases with the number of parties and with returns to rent-seeking efforts up to the point at which full dissipation occurs. At that point the social cost of rent-seeking activities equals the value of the rent. Interestingly, total expenditures in rent-seeking begin to decline after such point, as the number of parties and/or the returns to effort increase, because parties will start using the exit option undertaking mixed participation strategies.

We considered the impact of competitors' mixed participation strategies in the different context of redistributive and productive rent-seeking games among N parties. We showed that, although rent dissipation increases with the number of potential participants and approaches the entire value of the rent, when participants are relatively weak, an increase in the number of contestants actually leads to a reduction in the total dissipation of rent when players are relatively strong.

When the number of potential contestants and/or the returns to rent-seeking effort increase, parties undertaking mixed-participation strategies would play less often, although they would increase their expenditures when choosing to participate. In this case, the random combination of mixed participation strategies may lead to situations where no player enters the contest. Here the sought-after rent would remain unexploited, with a lost-treasure effect that may increase the social loss occasioned by rent-dissipation.

From a welfare point of view, whether unexploited rents should be computed among the social cost of rent-seeking obviously depends on the nature of the situation. In redistributive rent-seeking situations, if no party participates, no redistribution would take place, but no social loss would result from it. In productive rent-seeking situations, instead, lack of participation would create a social loss (the "lost treasure" effect), since valuable rents would be left unexploited.

We further showed that in N-party rent-seeking contests the lost-treasure effect perfectly counterbalances the reduction in rent dissipation due to competitors' exit. By computing how the sum of the parties' expenditures and the lost-treasure losses vary with a change in the number of players and returns to effort, we can in fact see that the sum of the expected values of these two costs always amounts to the full value of the rent. These results allow us to consider the overall impact of an increase in the number of potential contestants on the aggregate social loss. Looking at the total social loss as the sum of rent-seeking expenditures and lost-treasure losses, we see that in redistributive rent-seeking games involving strong contestants, the total social loss always decreases as the number of players increases. In productive rent-seeking situations the total social loss remains instead equal to the rent even when parties grow stronger, irrespective of their number.

These results have interesting policy implications. In redistributive games, an increase in the number of potential contestants reduces each player's incentive to enter the contest, decreasing the deadweight loss from dissipation. It is interesting to think that, by encouraging widespread participation in a redistributive game, a reduction in the social waste can be promoted.[20]

In productive rent-seeking situations, a change in the number of contestants alters the balance between the rent-dissipation and the lost-treasure components of the social loss. An increase in the number of contestants would discourage participation and leave potential value unexploited. Whenever the social value of the treasure is higher than the private value (e.g., the case of a scientific discovery that may have a social value greater than the private benefit captured by the discoverers), the social cost derived from the lost treasure would exceed the social benefit obtainable by a reduction in rent-dissipation. In these situations a reduction in the number of competitors in the research race may lead to greater opportunities for scientific discovery. These examples are illustrative of the important implications of our results and of the need to extend the analysis to additional settings with asymmetric rent-seeking parties and endogenous rent values, in order to assess their real scope for public policy and institutional design.

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Appendix

PROOF OF PROPOSITION 1

In the following, we provide a proof of the existence of pure *versus* mixed strategies equilibria. Straightforward but cumbersome calculations are required in order to prove the uniqueness of such equilibria. Thus, some of the details are omitted here; a complete proof is available upon request.

To begin with, note first that, for any i, when $q = 0$ then:

$$(A.1) \quad : \quad \frac{\partial U_i}{\partial X_i}(X_i, p_i; Y, q)_{|q=0} = -p_i$$
$$(A.2) \quad : \quad \frac{\partial U_i}{\partial p_i}(X_i, p_i; Y, q)_{|q=0} = 1 - X_i$$

while for $q = 1$ we obtain:

$$(A.3) \quad : \quad \frac{\partial U_i}{\partial X_i}(X_i, p_i; Y, q)_{|q=1} = \frac{rX_i^{-1}(N-1)}{N^2} - 1$$
$$(A.4) \quad : \quad \frac{\partial U_i}{\partial p_i}(X_i, p_i; Y, q)_{|q=1} = \frac{1}{N} - X_i$$

Now, assume that $r \leq \frac{N}{N-1}$; we can show that the profile where all players choose $\left(p = 1, X = r\frac{N-1}{N^2}\right)$ is Nash. In fact, if player i anticipates that the others always enter with certainty $\left(q = 1, Y = r\frac{N-1}{N^2}\right)$, he has no incentive to deviate since any strategy associated to a $p_i < 1$ and a smaller effort $X_i \leq r\frac{N-1}{N^2}$ by condition (A.3) and (A.4) leads to:

$dU_i(X_i, p_i; Y, q)_{|\left(q=1, Y=r\frac{N-1}{N^2}\right)} = \frac{1}{N}\left(1 - r\left(\frac{N-1}{N}\right)\right) dp_i \leq 0$, since under $r \leq \frac{N}{N-1}$ we have $1 - r\left(\frac{N-1}{N}\right) \geq 0$ and $dp_i < 0$. Thus, deviating is not profitable.

We now show that for $r > \frac{N}{N-1}$ a corner solution cannot be a Nash equilibrium. A profile of strategies where all players choose $(p = 0, X = 0)$ cannot be Nash: if player i anticipates that the others do not enter ($q = 0$,$Y = 0$), he has an incentive to deviate, that is he may enter more frequently and undertake a small positive effort (*i.e.* there exist a $p_i > 0$ and a $0 < X_i < 1$) such that by condition (A.1) and (A.2): $dU_i(X_i, p_i; Y, q)_{|(q=0, Y=0)} = dp_i - p_i dX_i > 0$. Thus, deviating is profitable.

Likewise, a profile where all players choose ($p = 1$,$X = r\frac{N-1}{N^2}$) cannot be Nash: if player i anticipates that the others always enter with certainty ($q = 1$,$Y = r\frac{N-1}{N^2}$), he has an incentive to deviate and choose a strategy with a lower $p_i < 1$ and a higher effort $X_i > r\frac{N-1}{N^2}$. By conditions (A.3) and (A.4): $dU_i(X_i, p_i; Y, q)_{|\left(q=1, Y=r\frac{N-1}{N^2}\right)} = \frac{1}{N}\left(1 - r\left(\frac{N-1}{N}\right)\right) dp_i + 0 dX_i = \frac{1}{N}\left(1 - r\left(\frac{N-1}{N}\right)\right) dp_i$

> 0, since, given $r > \frac{N}{N-1}$, we always have $\frac{1}{N}\left(1 - r\left(\frac{N-1}{N}\right)\right) < 0$ and $dp_i < 0$. Hence, deviating is profitable.

Thus, any Nash equilibrium in mixed strategies must have $0 < X < r\frac{N-1}{N^2}$ and $0 < p < 1$ which satisfy (4) and (5). We now prove that conditions (4) and (5) intersect at least once, implying the existence of at least one Nash equilibrium satisfying (4)-(5). Consider first condition (4); it implies that if $p = 0$, then $X = 1$, while if $p = 1$, then $X = \frac{1}{N}$. Consider now condition (5); it implies that if $p = 0$, then $X = 0$, while if $p = 1$, then $X = \frac{r(N-1)}{N^2} > \frac{1}{N}$ when $r > \frac{N}{N-1}$, yielding that (4) and (5) intersect at least once.

Concerning the uniqueness of this Nash equilibrium, the complete proof of the result requires that conditions (4)-(5) be rewritten. Since these alternative expressions help us solve later on the comparative statics analysis, we briefly sketch these calculations in the following. Using the same calculations as those introduced for the rent dissipation, it is easy to see that condition (4) may be written as:

$$\begin{aligned}(A.5) \quad : \quad X &= \frac{1}{Np}\sum_{j=1}^{N}\binom{N}{j}p^j(1-p)^{N-j} \\ &= \frac{1}{Np}(1-(1-p)^N)\end{aligned}$$

thus according to (A.5), $\forall p \in [0,1]$:[21]

$$\frac{\partial X}{\partial p} = \frac{1}{Np^2}\left(Np(1-p)^{N-1} + (1-p)^N - 1\right) < 0$$

Secondly, it can be shown that the relationship between X and p corresponding to condition (5) may be written as:

$$\begin{aligned}(A.6) \quad : \quad X &= \frac{r}{Np}\left[\sum_{j=1}^{N}\binom{N}{j}p^j(1-p)^{N-j}\left(\frac{j-1}{j}\right)\right] \\ &= \frac{r}{Np}\left[\sum_{j=1}^{N}\binom{N}{j}p^j(1-p)^{N-j} - \sum_{j=1}^{N}\binom{N}{j}p^j(1-p)^{N-j}\frac{1}{j}\right] \\ &= \frac{r}{Np}(1-(1-p)^N) - rH(p)\end{aligned}$$

where $H(p)$ is given by:[22]

[21] Notice that this implies that condition (4) exhibits a monotonous decreasing relationship between X and p, in the interval $[0,1]$ for p.

[22] We are indebted to Bruno Lovat for this result.

$$H(p) = \int_1^0 (px + (1-p))^{N-1} Ln(x) dx$$

Notice also that, since

$$H'(p) = (N-1) \int_1^0 (x-1)(px + (1-p))^{N-2} Ln(x) dx$$

with x taking value on $[0, 1]$, it is clear that $H'(p) < 0$, which is used thereafter.

To illustrate the uniqueness of the Nash equilibrium, we performed several simulations for the first order conditions (4) and (5), choosing several values for $r > \frac{N}{N-1}$ and N. The following graphics plot the results of the simulations for $r = 5$, setting different values for $N \in \{3, 10, 100\}$.

INSERT FIGURE 4

It is evident that equations (4) and (5) display a unique intersection point, thus suggesting that there exist a unique Nash equilibrium of the game.

COMPARATIVE STATICS FOR r

We will first prove that $\frac{\partial p}{\partial r} < 0$ and $\frac{\partial X}{\partial r} > 0$. Totally differentiating (4)-(5) or equivalently (A.5)-(A.6) leads to the system:

$$(A.8) \quad : \quad \frac{\partial X}{\partial r} = \frac{\partial p}{\partial r} \frac{1}{Np^2} \left(Np(1-p)^{N-1} + (1-p)^N - 1 \right)$$

$$(A.9) \quad : \quad \frac{\partial X}{\partial r} = r \frac{\partial p}{\partial r} \left(\frac{1}{Np^2} \left(Np(1-p)^{N-1} + (1-p)^N - 1 \right) - H'(p) \right) + \left(\frac{1}{Np} (1 - (1-p)^N) - H(p) \right)$$

where $H(p)$ and $H'(p) < 0$ have been previously defined in the proof of proposition 1. Substituting (A.8) in (A.9), we obtain:

$$\frac{\partial p}{\partial r} = \frac{\left(\frac{1}{Np} (1 - (1-p)^N) - H(p) \right)}{(1-r) \left(\frac{1}{Np^2} \left(Np(1-p)^{N-1} + (1-p)^N - 1 \right) \right) + rH'(p)} < 0$$

The denominator is negative, while the numerator (which is equal to X/r according to (A.6)) is positive: as a result $\frac{\partial p}{\partial r} < 0$. Then, given (A.8), it is clear that:

$$sign \frac{\partial X}{\partial r} = -sign \frac{\partial p}{\partial r} > 0$$

Now, we show that $\frac{\partial T}{\partial r}(r,N) = -\frac{\partial D}{\partial r}(r,N) > 0$.

Consider that $T(r,N) = (1-p)^N$; the impact of an increase in r on the lost treasure is thus:

$$\frac{\partial T}{\partial r}(r,N) = N(1-p)^{N-1}\left(-\frac{\partial p}{\partial r}\right) > 0$$

Thus, by $D(r,N) = 1 - T(r,N)$, an increase in r has a negative impact on the rent dissipation: $\frac{\partial D}{\partial r}(r,N) \leq 0$.

TABLE 1 - *comparative statics for r (results for N = 3)*

r	X	p	D	T
2	.4375	.75	.9843	.0157
3	.5622	.5321	.8975	.1025
4	.6408	.4170	.8019	.1981
5	.6955	.3439	.7175	.2825

COMPARATIVE STATICS FOR N

Since N is discrete, the analysis of its impacts is quite complex, making it useful to reconsider the simulations already performed. Looking at figure 4, it is evident that as N increases the equilibrium values of p and X (the intersection point of (4) and (5)) shifts down and to the left, implying a decrease in both p and X.

On the other hand, a more informal argument may be helpful and sufficient to describe the effects on the equilibrium strategy. First, in proposition 1, we have seen that there exists a higher bound for the effort X associated with the equilibrium in mixed strategy. This bound is $\frac{N-1}{N^2}r$ the efficient effort when players never randomize, which is decreasing in N. Thus, X cannot increase in N.

Second, remember that for each participant, the number of his opponents is distributed according to a Binomial law, with by definition a probability to face j opponents equal to $\Pr(n=j) = \binom{N-1}{j}p^j(1-p)^{N-1-j}$; it can be shown that:

$$\Pr(n>k) = \sum_{j=k+1}^{N-1}\binom{N-1}{j}p^j(1-p)^{N-1-j} = \frac{(N-1)!}{k!(N-k-2)!}\int_0^p t^k(1-t)^{N-k-2}dt$$

meaning that $\Pr(n>k)$, the cumulative probability that the number of parties playing the game be higher than a threshold k, is monotonously increasing in N. As a result, an increase in N means an increase in the risk borne by each party (in the sense of the first stochastic dominance order) to share the constant

prize with a greater number of opponents - the higher the number of players, the smaller the individual share $\left(\frac{1}{1+j}\right)$. To compensate, each party reduces his probability of participation. Thus, the equilibrium value of p decreases with N.

TABLE 2 - *comparative statics for* N (*results for* $r = 2$)

N	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*20*	*50*	*100*
X	.4375	.4154	.4040	.3970	.3923	.3889	.3863	.3843	.3757	.3710	.3694
p	.75	.5837	.4753	.4000	.3453	.3036	.2708	.2443	.1235	.0496	.0248
D	.9843	.9699	.9602	.9534	.9484	.9447	.9417	.9393	.9284	.9217	.9195
T	.0157	.0301	.0398	.0466	.0516	.0553	.0583	.0607	.0716	.0783	.0805

FIGURES

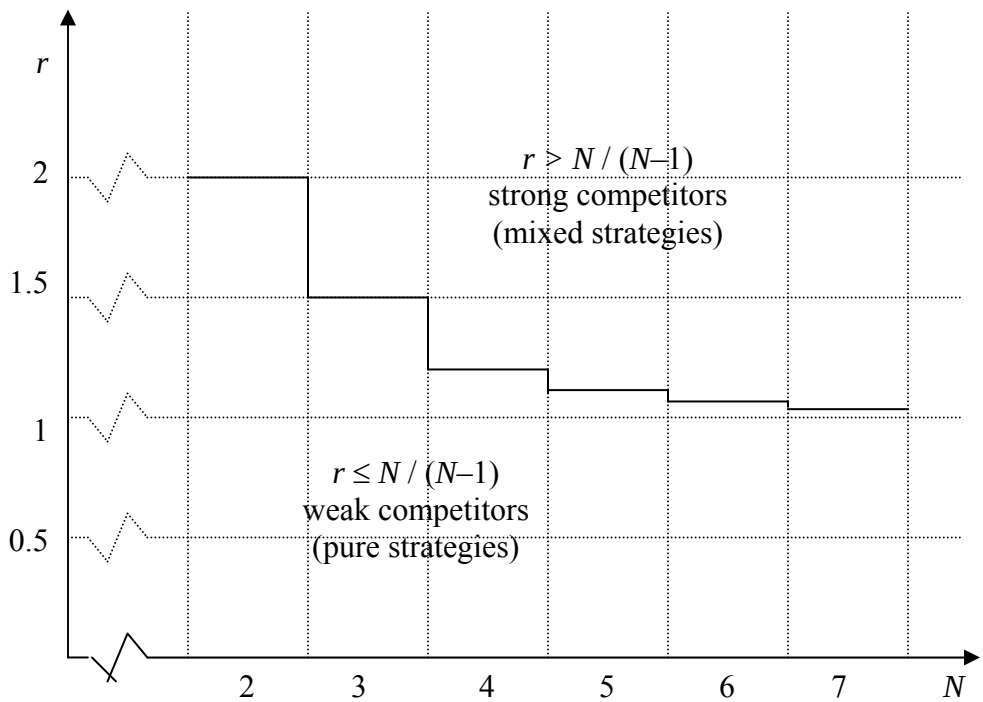

FIGURE 1: Parties' strength as a function of r and N

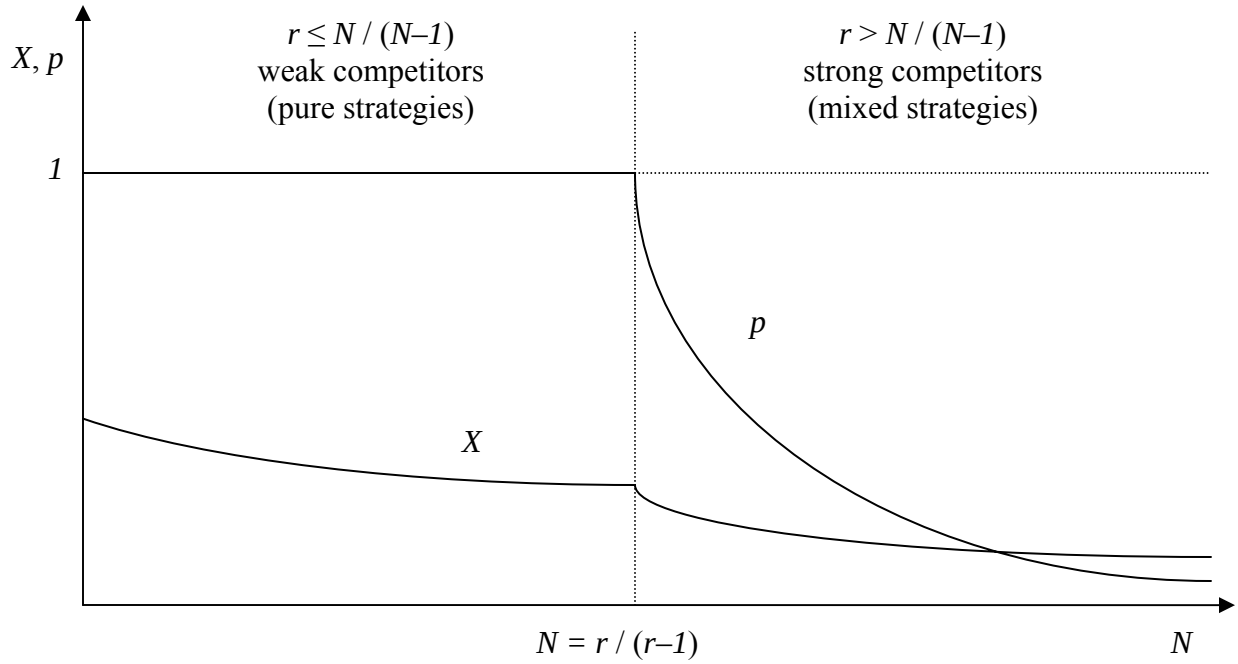

FIGURE 2: Rent-seeking expenditure X and probability of participation p as a function of the number of contestants N (for any r)

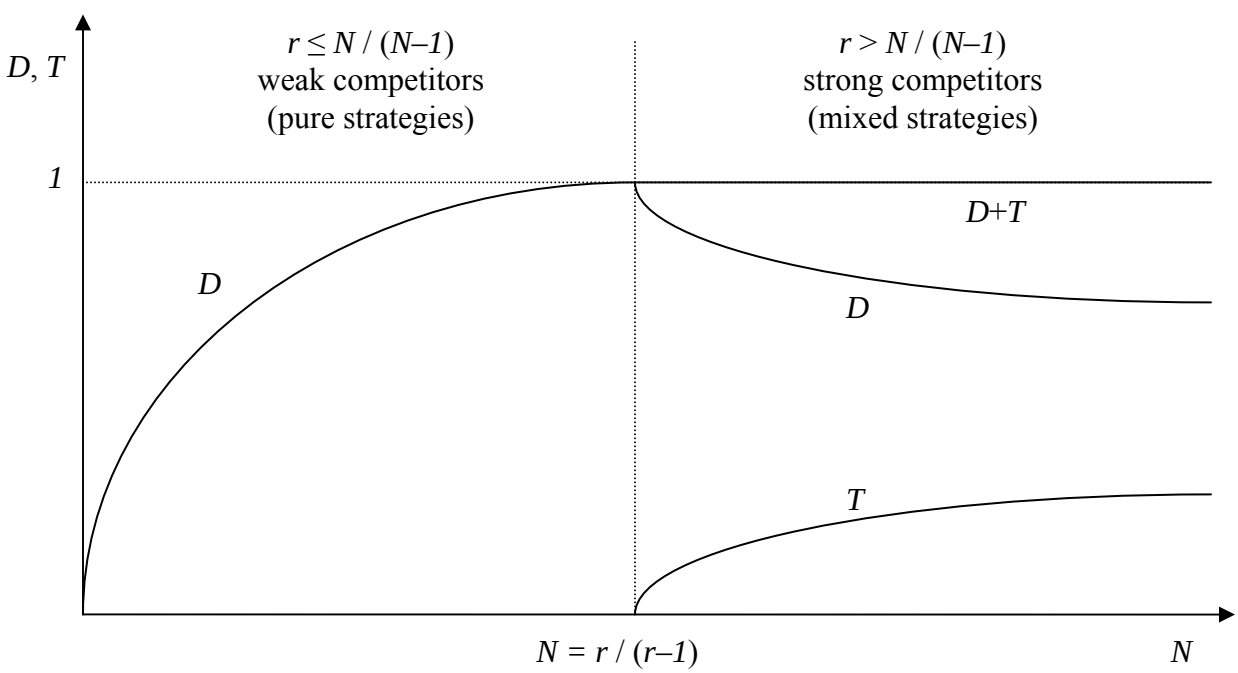

FIGURE 3: Rent dissipation D and lost treasure T as a function of the number of contestants N (for any r)

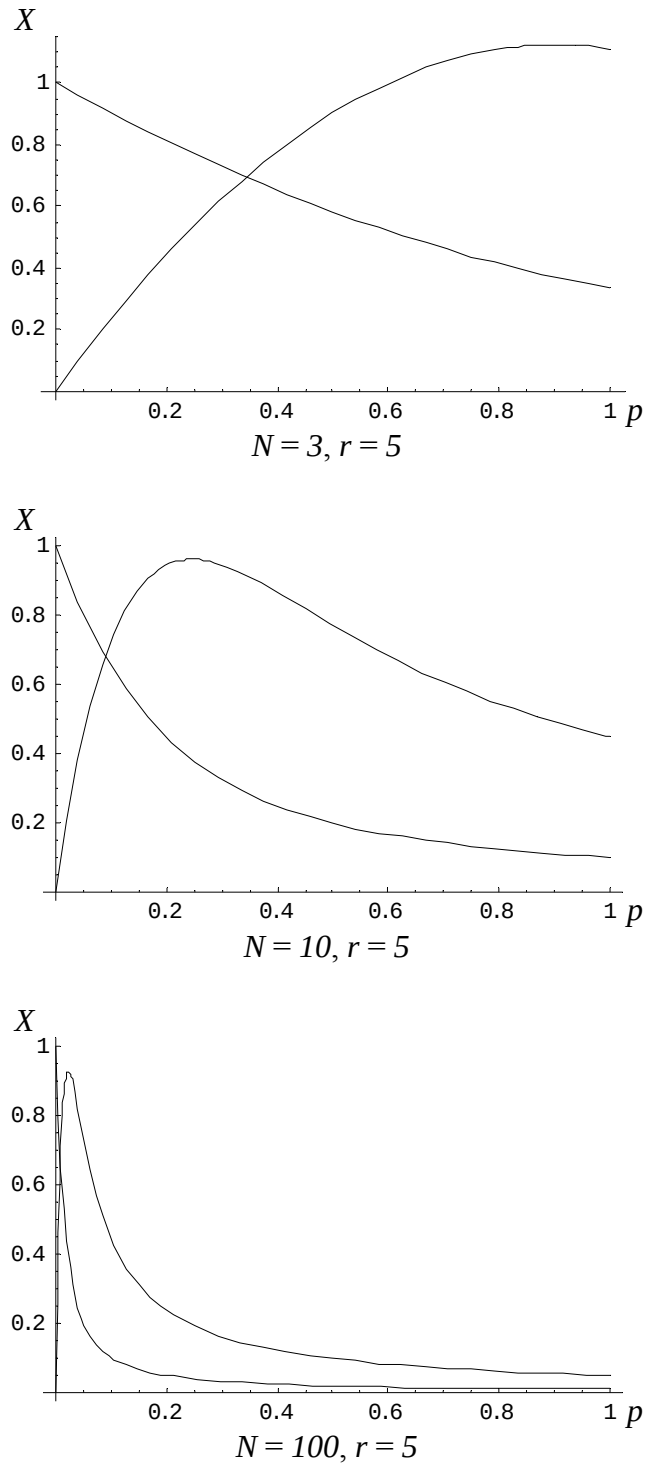

FIGURE 4: Simulation of the first order conditions (4) and (5)
(Condition (4) is the monotonically decreasing curve in each plot)