When Will Judgment Proof Injurers Take Too Much Precaution?

Giuseppe Dari Mattiacci\textsuperscript{a, b}, Gerrit De Geest\textsuperscript{a, *}

\textsuperscript{a} Utrecht School of Economics, Vredenburg 138, 3511 BG Utrecht, The Netherlands
\textsuperscript{b} George Mason University, School of Law, 3301 North Fairfax Drive, Arlington, Virginia 22201, USA

Abstract

This article identifies the conditions under which insolvent injurers over-invest in precaution. We show that this may happen only when precaution reduces the probability of the accident. No such overprecaution occurs if precaution only reduces the magnitude of the harm.

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\textsuperscript{*} Corresponding author: Phone 0031 (0)30 253 9800, Fax 0031 (0)30 253 7373. E-mails: g.darimattiacci@econ.uu.nl (Giuseppe Dari Mattiacci), g.degeest@econ.uu.nl (Gerrit De Geest).

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1. Introduction: one-pocket v. two-pocket models

An injurer is judgment proof if his assets are less than the harm. Summers (1983) and Shavell (1986) showed that judgment proof injurers tend to take too little precaution because not all accident losses are internalized. In order to prove this result, Shavell (1986) used the standard probability model, in which precaution reduces the probability but not the magnitude of accidental harm. In addition, Shavell made the simplifying assumption that the injurer’s precaution expenses do not reduce the assets available for compensation. We refer to this model as a two-pocket model because the injurer behaves as if he had two separate pockets: one limited, for liability payments, and another unlimited, for precaution. Beard (1990) relaxed this assumption and showed that in a one-pocket probability model (in which precaution and liability expenses are paid out of the same pocket) the injurer may take overprecaution under certain conditions.

Building on our previous contribution,1 this paper further refines the analysis by making a distinction between precaution that reduces the probability and precaution that reduces the magnitude of the accidental harm. We study the pattern of the injurer’s precaution decision under different models and show a general result: the overprecaution effect arises only for those precautionary measures that reduce the probability of accidents.

2. Probability models

Accidents occur under strict liability between a passive victim and an injurer, strangers to each other. All functions are continuously differentiable to any desired order. Let:

\[ x = \text{the injurer’s precaution cost}, \quad x \geq 0; \]

\[ p(x) = \text{probability of an accident}, \quad 0 < p(x) < 1, \quad p' < 0, \quad p'' > 0; \]

\[ h = \text{magnitude of the harm}, \quad h > 0; \]

\[ t = \text{the injurer’s assets}, \quad t > 0. \]

We employ the following social cost function:

\[ S(x) = p(x)h + x \quad (1) \]

Let \( x^* \) denote the (unique) level of precaution that minimizes \( S(x) \) and let it be positive. The injurer chooses the level of precaution that minimizes the sum of expected liability and precaution cost.

**Proposition 1.** In a two-pocket probability model:

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1 Dari Mattiacci and De Geest, forthcoming.
(1.I) The injurer takes \( x^* \) if \( t \) is equal to or greater than \( t_2 = h \);
(1.II) Otherwise, he takes \( x_2 < x^* \), which increases continuously in \( t \).

In a one-pocket probability model:
(1.III) The injurer takes \( x^* \) if \( t \) is equal to or greater than \( t_1 \), where \( t_1 > h + x^* \);
(1.IV) Otherwise he takes \( x_1 \), which increases continuously in \( t \);
(1.V) As \( t \) increases, \( x_1 \) is initially less than, then equal to and finally greater than \( x^* \);
(1.VI) As \( t \) increases, \( x_1 \) is initially less, than equal to and finally greater than \( x_2 \).

**Proof:** We employ the following algorithm solution: a) Find the levels of \( x \) that minimize the total expenditures for a solvent and an insolvent injurer – a marginal analysis; b) Compare the total expenditures and choose whether to be solvent or insolvent – an inframarginal analysis; c) Verify that this is always a valid solution, that is, that the injurer is actually solvent (insolvent) at the chosen levels of precaution.

In a two-pocket probability model, the injurer’s expenditure function is:

\[
\begin{align*}
J(x) &= p(x)h + x & \text{if } h \leq t \\
J_2(x) &= p(x)t + x & \text{if } h > t
\end{align*}
\]

(2)

Let \( x_2 \) denote the level of \( x \) that minimizes \( J_2(x) \) and let it be positive; \( x^* \) minimizes \( J(x) \). The solution algorithm trivially applies and claims (1.I) and (1.II) are self-evident.

In a one-pocket probability model, the insolvent injurer pays compensation equal to \( t-x \). His expenditure function is:

\[
\begin{align*}
J(x) &= p(x)h + x & \text{if } h + x \leq t \\
J_1(x) &= p(x)[t-x] + x & \text{if } h + x > t
\end{align*}
\]

(3)

Let \( x_1 \) denote the level of \( x \) that minimizes \( J_1(x) \), and let it be positive. The injurer takes \( x^* \) if \( J(x^*) \leq J_1(x_1) \). He takes \( x_1 \), otherwise. Thus, \( x^* \) is a solution iff:

\[
t \geq \left\{ p(x^*)h + x^* - \left[ 1 - p(x_1) \right] x^*_1 \right\} / p(x_1)
\]

(4)

Claim (1.III): by the Envelop Theorem, \( dJ_1(x_1)/dt > 0 \). Thus, since \( J(x^*) \) is constant in \( t \), there exists a unique \( t_1 \) equal to the right-hand side of Exp. (4) such that the injurer always takes \( x^* \) if \( \geq t_1 \) and \( x_1 \), otherwise. If \( t = h + x^* \), then \( J(x^*) = J_1(x^*) > J_1(x_1) \); thus, \( t_1 \) must be greater than \( h + x^* \) for \( J(x^*) < J_1(x_1) \). Claim (1.IV): by the Implicit Function Theorem on the f.o.c. for \( J_1(x) \), \( dx_1/dt > 0 \). Claim (1.V): Assume \( t = h + x^* \). Evaluating

\footnote{In Beard (1990), the injurer’s precaution is not necessarily increasing in its assets because of the random distribution of the harm. In our model, this ambiguity is sharpened into an increase above the optimal level and then a sudden drop to the optimal value.}
the first derivative of \( J_1(x) \) at \( x^* \) we obtain \( p'(x^*)[t-x^*]+1-p(x^*)<0 \), because the first two terms amount to zero by the f.o.c. for \( J(x) \), and the third term is negative; thus, \( x_1>x^* \). It can be shown that when \( t \) approaches 0, \( x_1 \) also approaches 0; thus, for initial levels of \( t \), \( x_1=x^* \) and then \( x_1>x^* \). Claim (1.VI): Evaluate the first derivative of \( J_1(x) \) at \( x_2 \) and note that \( x_1\leq x_2 \) if \( p(x_2)+p'(x_2)x_2\leq0 \), which can be interpreted as a condition depending on the elasticity of the probability function or – given \( p'(x_2)=-1/t \) – it can be rewritten as \( t\leq x_2/p(x_2); x_1>x_2 \), otherwise. At \( t=h \) the criterion becomes \( h\leq x^*/p(x^*) \); therefore \( x_1 \) crosses \( x_2 \) to the left of \( t=h \) (as in figure 2) if \( x^*<p(x^*)h \) – the cost of precaution is less than the expected accident loss at the social optimum –, \( x_1 \) crosses \( x_2 \) at (or to the right of) \( t=h \), if \( x^*\geq p(x^*)h \).

Finally, to verify point c) above, if \( x^* \) is chosen, the injurer must actually be solvent at \( x^*(h+x^*\leq t) \), as implicitly required by Exp. (3). Assume the solution is \( x^* \) and, contrary to our claim, \( h+x^*>t \). Then we could write \( p(x^*)h+x^*>p(x^*)[t-x^*]+x^*>p(x^*)[t-x^*]+x^* \) (by definition of \( x_1 \)). This would imply \( J(x^*)>J(x_1) \) and, thus, the solution would be \( x_0 \), which contradicts the premise. Therefore, if \( x^* \) is the solution, then \( h+x^*\leq t \) must be satisfied. A similar contradiction arises if \( x_1 \) is chosen and \( h+x_1>t \) is not satisfied. Hence, when the injurer chooses an inefficient level of precaution he is actually insolvent at that level. Q.E.D.

### 3. Magnitude models

Modifying the previous setting, let:

\[
\begin{align*}
 p &= \text{probability of an accident, } 0<p<1; \\
 h(x) &= \text{magnitude of the harm, } h(x)>0, h'&<0, h''>0.
\end{align*}
\]

The social cost function is:

\[
S(x) = ph(x) + x
\]  
(5)

Let \( x^* \) denote again the socially optimal level of precaution.

**Proposition 2.** In a two-pocket magnitude model:

(2.I) The injurer takes \( x^* \) if \( t \) is equal to or greater than \( t\geq h(x^*)+x^* \);

(2.II) Otherwise, he takes \( x_2=0 \).

In a one-pocket magnitude model:

(2.III) The injurer takes the same levels of precaution as in the two-pocket version (either \( x^* \) or \( x_1=x_2=0 \) at the same conditions \( t_1=t_2 \)).

**Proof:** In a two-pocket magnitude model, the injurer’s expenditure function is:

\[
\begin{align*}
J(x) &= ph(x) + x \quad \text{if } h(x) \leq t \\
J_2(x) &= pt + x \quad \text{if } h(x) > t
\end{align*}
\]  
(6)
$J_2(x)$ is minimized by $x_2=0$. The injurer takes $x^*$ if $J(x^*) \leq J(t(0))$. He takes $x_2=0$ otherwise. Thus, $x^*$ is taken iff:

$$t \geq h(x^*) + x^*/p$$

(7)

There exist a unique $t_2$ equal to the right-hand side of Exp. (7) such that the injurer always takes $x^*$ if $t \geq t_2$ and $x=0$ otherwise. Point c) is proven as in section 0.

In a one-pocket magnitude model, the injurer’s expenditure function is:

$$\begin{align*}
J(x) &= ph(x) + x & \text{if} & \quad h(x) + x \leq t \\
J_1(x) &= p[t - x] + x & \text{if} & \quad h(x) + x > t
\end{align*}$$

(8)

$J_1(x)$ is minimized by $x=0$. Since $J_1(0)=J_2(0)$, the injurer takes the same levels of precaution as in the previous model. Q.E.D.

4. Mixed probability-magnitude models

If the same precaution reduces the probability and the magnitude of the harm (joint-probability-magnitude model) at the same time, the social cost is:

$$S(x) = p(x)h(x) + x$$

(9)

It can be shown that the one-pocket version yields similar results as the one-pocket probability model.

If two independent precautionary measures, $s$ and $z$, reduce the probability and the magnitude of the harm, respectively (separate-probability-magnitude model), the social cost is:

$$S(s, z) = p(s)h(z) + s + z$$

(10)

It is easy to show that both the one- and the two-pocket version may yield overprecaution only with respect to $s$. In addition:

**Corollary 1.** In a one-pocket separate-probability-magnitude model, an insolvent injurer might spend more in total for $s+z$ than a solvent injurer ($s_t > s^* + z^*$).

**Proof:** Consider $t = h(z^*) + s^* + z^*$, and proceed as in Proposition 1, (1.V). Q.E.D.

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3 Dari Mattiacci and De Geest (forthcoming) show that overprecaution in a two-pocket separate-probability-magnitude model – where there is no precaution subsidy – is due to a substitution effect between probability-precaution and magnitude-precaution.
5. Logic and implications of the results

To understand why overprecaution may only concern probability-precaution, consider that, on the one hand, judgment proofness provides the injurer with an implicit harm-subsidy since a portion of the harm is externalized on the victim. On the other hand, (only in one-pocket models) judgment proofness provides the injurer with an implicit precaution-subsidy: the more the injurer spends on precaution, the less remains available for compensation.

These implicit subsidies have opposite effects on the incentives to take precaution: the former reduces them while the latter reinforces them. However, while the precaution-subsidy has comparable effects under all models, the harm-subsidy is weaker in probability models – where it simply induces a lower level of precaution – than in magnitude models – where it results in no precaution. For this reason, the harm-subsidy always prevails on the precaution-subsidy in magnitude models and overprecaution never occurs.

In addition, one-pocket models also impose an upper limit on precaution, as precaution costs cannot exceed the assets. Consequently, for low assets, precaution taken in a one-pocket model may be lower than in a two-pocket model, despite the precaution subsidy (see claim (1.VI)). Figure 1 clearly depicts our results.

We also suggest that overprecaution might arise even if the accident is particularly unlikely and the expenditure on precaution is negligible in relation to the harm, and even when the injurer would have been solvent had he taken optimal precaution (see claims (1.III) and (2.I)).

Many potentially harmful activities are subject to regulation. One of the major justifications for regulatory intervention is the concern that tort law alone would fail to enhance optimal precaution because injurers are judgment proof. Our analysis shows that it is important to distinguish between different categories of accidents.

6. References


FIGURE 1: Levels of precaution in the probability model and in the magnitude model

One-pocket model  Two-pocket model

**PROBABILITY MODEL**

**MAGNITUDE MODEL**