Judgment Proofness under Four Different Precaution Technologies
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Abstract
This study shows that the effects of judgment proofness on precaution depend on whether the injurer can reduce the probability of the accident, the magnitude of the harm, or both. Different legal solutions to the problem are examined: punitive damages, average compensation, undercompensation, accurate compensation and negligence. We find that when the injurer can only reduce the probability of the accident, negligence with average compensation is the best solution, but negligence with perfectly compensatory damages is the desirable solution if the injurer can only or also affect the magnitude of the harm.

Keywords: insolvency, judgment proof, liability, bankruptcy

JEL classification: K13, K32

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1. Introduction

An injurer is potentially judgment proof if he can cause more harm than he is able to compensate. Judgment proofness derives from a liability threshold on the injurer caused by his limited assets or by the law – like in cases of corporate limited liability or liability caps.\(^1\) In principle, potentially judgment proof injurers can be expected to take less than optimal precaution since they bear only a part of the accident loss.

The incentive effects, however, largely depend on the precaution technology. This article tries to draw a more general picture than previous contributions. The literature has mainly been based on one type of model, in which more precaution reduces the probability of an accident but not the magnitude of the harm. We call this model the *probability model*. It gives a realistic picture for some accident types, like an air crash, where more precaution of the pilot leads to fewer accidents but the harm in the case of an accident remains constant. Yet, in most real-world situations, the injurer can also influence the magnitude of the externality.

Therefore, we also analyze three additional models, in which the injurer has some degree of control over the magnitude of the harm. The (pure) *magnitude model* depicts situations in which the injurer’s precaution reduces only the magnitude of the harm, while the probability of the accident is exogenous. In most nuisance cases, for instance, less precaution (e.g. more noise at night) leads to more serious externalities to the neighbors. The *joint-probability-magnitude model* portrays cases in which the injurer’s precaution simultaneously reduces the probability of an accident and the magnitude of the resulting harm; as is the case, for instance, when a motorist reduces his speed. Finally, the *separate-probability-magnitude model* refers to accidents in which injurers can take one precautionary measure to reduce the probability of an accident and another precautionary measure to reduce the magnitude of the harm. For instance, the owner of a ship can invest in advanced navigation devices to reduce the probability of a shipwreck. However, once a shipwreck has occurred, the presence of lifeboats on board will enormously reduce the magnitude of the damages to the passengers and to the crew – although lifeboats are clearly irrelevant for the occurrence of the accident.

There is a fundamental difference between the outcome under a probability model and under a magnitude model. While under the probability model potentially judgment proof injurers systematically take too little precaution, injurers may take optimal precaution or no precaution at all under the magnitude models.

\(^1\) Removing the cap is an obvious solution that we will not discuss since our aim is to analyze the effects and not the reasons of such thresholds.
The intuition behind this binary outcome is that when the injurer can influence the magnitude of the harm, he can, in essence, decide whether or not to be potentially judgment proof (by causing harm that exceeds his assets). If he decides not to be potentially judgment proof (and to situate himself in the solvent zone), he internalizes the full harm, and hence, he will take optimal precaution. If he decides to be potentially judgment proof (and to be in the judgment proof zone), his liability expenses in the case of an accident will always equal his total assets. Since in the judgment proof zone taking more precaution does not reduce his liability expenses, he will choose no precaution at all, as more precaution would increase his precaution costs without reducing his exposure to liability. Therefore, the injurer’s decision is simply whether to be potentially judgment proof (no precaution) or to be solvent (optimal precaution).

In the separate-probability-magnitude model, there is an identical binary outcome with respect to the magnitude-reducing precaution, and the outcome with respect to probability-reducing precaution depends on the former outcome. If the injurer has chosen optimal magnitude-reducing precaution, then he chooses optimal probability-reducing precaution as well (as he remains solvent). However, if the injurer has decided to choose no magnitude-reducing precaution at all (and hence to enter the judgment proof zone), we find an intriguing result: it is possible that the injurer takes probability-reducing overprecaution. The intuition behind this result is as follows. If the injurer decides not to invest in magnitude-reducing precautionary measures (like safety belts, helmets, and lifeboats), he saves on precaution costs, but his liability expenses in the case of an accident may be higher than optimal. Accidents cost the injurer more, and he may thus choose to spend more on preventing them.

We will study the equilibrium level of precaution taken by the injurer in each of these four models in a formal framework and consider the negligence rule and non-compensatory damages as possible means to improve accident prevention. Table 1 and the propositions may provide the reader with a detailed summary of our results.

Several aspects of liability thresholds have already been studied in the law and economics literature, although under different labels (insolvency, liability caps, corporate limited liability, bankruptcy, inability to pay damages). In this article, we consider all these different issues as being of the same nature and label them as judgment proof problems for the sake of generality.² Our analysis applies to each of these problems and can be extended to

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² The case of disappearing defendants (where there is a certain probability that the injurer will not be sued) does not generate a maximum upper threshold on the damages but, rather, dilutes the probability of paying damages. Under a probability model, the two problems have the same effect on the injurer’s incentives, as both reduce the portion of the harm that the injurer internalizes (see Shavell (1984), who considers the two together). However, under magnitude models, they generate different effects. If there is judgment proofness, the injurer pays the minimum between the harm and the threshold: the portion of the harm that exceeds the threshold is completely externalized. On the contrary, if there is a probability a that the injurer will not be sued, he internalizes an accident
contract liability as well since the parties’ incentives are also the main concern in contract settings.

The first formal analyses of the judgment proof problem were made by Summers (1983), Landes and Posner (1984) and Shavell (1986); these analyses can be referred to as probability models. Summers (1983) first examined negligence as a corrective mechanism for the judgment proof problem and also discussed the case of insolvency among multiple tortfeasors. Landes and Posner (1984) studied this problem in connection to accidents involving a number of victims. Shavell (1986) examined both negligence and insurance.

Boyd and Ingberman (1994) analyzed the judgment proof problem by employing a probability model, and showed that punitive damages can be a solution.\(^3\) They also considered a simple case in which the magnitude of the harm can be reduced by means of precaution, and they proposed undercompensation as a solution. We argue that the best solution for the probability model is negligence with average compensation and provide a more general analysis of the magnitude models\(^4\) concluding that the best solution in these three models is the negligence rule with perfectly compensatory damages.

This paper is structured as follows. In section 2, we provide a formal analysis of the judgment proof problem under the four models. In section 3, we discuss the possible solutions to the judgment proof problem. Section 4 concludes. Some of the proofs are dealt with in the appendixes.

\[\text{Table 1}\]

2. The model

In this section, we give a formal interpretation of the four models described in the introduction. We make the following assumptions:

1. We are dealing with unilateral-precaution, unilateral-risk situations. That is, only the

loss lower than the actual harm, \(ah(x)\), for any level of \(x\). Hence, the case of disappearing defendants has the same effect on all models, and cannot be considered as a maximum upper threshold on liability. A punitive-damage coefficient equal to \(1/ah\) solves the problem under all models. It is finally important to remark that, on the terminological side, ‘disappearing defendant’ and ‘judgment proof injurer’ had identical meanings in Summers (1983) and Shavell (1986), as they refer to both the inability to pay for damages and the possibility of not being sued without further distinction. We prefer distinguishing between the two and (somewhat arbitrarily) using ‘disappearing defendant’ to refer to the possibility for the injurer not to be sued and ‘judgment proofness’ to refer to an upper threshold on the injurer’s liability.

\(^3\) Lewis and Sappington (1999) also consider non-compensatory damages as a solution to the judgment proof problem but do not distinguish between different models as we do. In Boyd and Ingberman (1999), the authors study the effect of punitive damages when the assets \(t\) are endogenously determined. In our analysis, \(t\) is exogenous.

\(^4\) Models that take into account the effect of the injurer’s precaution on the magnitude of the accident loss have occasionally been employed in the literature in different contexts from ours. See for example Spier (1994), in which a (technically different from ours) separate-probability-magnitude tort model is employed in analyzing the effects of accurate and inaccurate damage awards on settlements bargaining, and Kornhauser and Revesz (1990),
injurer can take precaution, and only the victim suffers harm;
2. The rule in force is strict liability;
3. The courts entitle victims to perfect compensation;
4. Parties are perfectly informed of the legal rules, risk-neutral, rational, and utility maximizing;
5. The precaution costs do not reduce the total assets available for paying compensation.  

Assumptions 2 and 3 are relaxed in section 3. General notation is:

\[ x, s, z = \text{the expenditure on precaution or its monetary equivalent if precaution is non-monetary}; \]
\[ x \geq 0, s \geq 0, z \geq 0; \]
\[ x \text{ will be used in the first three models, while in the fourth model we will make use of } s \text{ and } z \text{ in order to distinguish between the probability-reducing precaution } s \text{ and the magnitude-reducing precaution } z; \]
\[ p = \text{the probability of an accident}, 0 < p < 1; \]
\[ h = \text{the magnitude of the harm if an accident occurs}, 0 < h \leq h^{\text{max}}, \text{where } h^{\text{max}} \text{ is finite}; \]
\[ J(\cdot) = \text{the injurer’s total costs (expected liability plus precaution)}; \]
\[ S(\cdot) = \text{the social cost}; \]
\[ t = \text{the maximum upper threshold (maximum amount of damages that the injurer can pay)}; 0 < t < h^{\text{max}} \text{ (the injurer is potentially judgment proof)}. \]

The social cost function is defined as the sum of the expected accident loss (the probability of an accident times the magnitude of the harm) and the expenditure on precaution.

2.1 The probability model

In this model, the probability of the accident is a function of the level of precaution taken by the injurer, while the magnitude of the harm varies exogenously.  

Let:
\[ p(x) = \text{the probability of an accident}, 0 < p(x) < 1, \quad p' < 0, \quad p'' > 0; \]
\[ h = \text{the magnitude of the harm}; \]
\[ f(h) = \text{the density function of } h, f(h) > 0 \text{ over } 0 < h \leq h^{\text{max}} \text{ and } f(h) = 0 \text{ elsewhere}, \quad \text{and } F(h) = \text{the cumulative distribution}; \]
\[ E(h) \]

in which a magnitude model is employed for the study of insolvency under joint and several liability.  

5 This simplifying assumption is also made in Shavell (1986). A model in which this assumption is made can be called a ‘two-pocket model’, since it is as if the injurer had two pockets: one for making precaution costs and one for paying compensation. Two-pocket models can be realistic when there is a legal liability cap (a legal upper limit on the amount of damages that the victim is entitled to recover) or when precaution is non-monetary. Beard (1990) was the first to analyze a (stochastic) one-pocket (probability) model; he showed that overprecaution could occur. In Dari Mattiacci and De Geest (2003), we show that this effect does not hold for all precaution technologies. Garoupa and Gravelle (2002) applies a similar refinement to the case of law enforcement with legal expenditures.

6 The probability-model closely follows Shavell (1986).
denote the average harm. $^7$

The social cost function is:

$$S(x) = p(x)E(h) + x$$

where the average harm $E(h)$ is given by $E(h) = \int_0^{h_{\text{max}}} f(h)dh$. Let $x^*$ denote the (unique) level of precaution that minimizes the social cost function, and assume it is positive (the same will apply to all models).

**Proposition 1**: In a probability model, a potentially judgment proof injurer systematically takes underprecaution. The lower the threshold, the lower the level of precaution that the injurer takes.

If a threshold $t$ is present, then the injurer pays compensation equal to the harm only if the harm is lower than or equal to the threshold; he pays compensation equal to the threshold otherwise. Thus, a potentially insolvent injurer only internalizes:

$$J_t(x) = p(x)E_t(h) + x$$

where $E_t(h) = \int_0^h f(h)dh + \int_t^{h_{\text{max}}} f(h)dh < E(h)$, which proves the first claim of the proposition.

The second claim follows from the fact that when $t$ rises, $E_t(h)$ also increases, and consequently, $x_t$ moves towards $x^*$. Figure 1 illustrates this result.

![Figure 1](image)

### 2.2 The magnitude model

In this model, the magnitude of the harm is a function of the level of precaution taken by the injurer, while the probability of the accident is exogenous. In such a model, the injurer is actually judgment proof only if he takes a level of precaution that results in a greater harm than the threshold. A potentially judgment proof injurer also decides whether or not to be solvent by choosing his level of precaution.

We shall examine the injurer’s behavior by employing a simple algorithm. We will first look for the level of precaution that minimizes the injurer’s total costs if he is solvent. Second,

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$^7$ The most simple probability model would be one where the magnitude of the harm is a constant. With such a model, however, the analysis of all tort law solutions that we analyze in the next section would become pointless. Therefore, we introduce some stochastic variations in the probability model. That is, harm is sometimes low and sometimes high, but the magnitude of the harm is always beyond the control of the injurer.
we will calculate the level of precaution that minimizes his total costs if he is insolvent. Third, we will compare the minimum total costs in the solvent zone to the minimum total costs in the judgment proof zone and find the injurer’s optimal decision of whether to be solvent or insolvent. Appendix 1 proves the validity of this algorithm, which also applies to the next two models.

Let:
\[ p \text{ = the probability of an accident, } 0 < p < 1; \]
\[ h(x) \text{ = the magnitude of the harm if an accident occurs, } 0 < h(x) \leq h^\text{max}, h' < 0, h'' > 0. \]

The social cost function is:
(3) \[ S(x) = ph(x) + x \]

Let \( x^* \) denote the level of precaution that minimizes the social cost function.

Proposition 2: In a magnitude model, if the threshold is sufficiently high — \( t \geq h(x^*) + x^*/p \) — the injurer takes optimal precaution. Otherwise, he takes no precaution at all.

The injurer’s cost function is:

(4) \[
\begin{align*}
J(x) &= ph(x) + x & \text{if } h(x) \leq t \\
J_i(x) &= pt + x & \text{if } h(x) > t
\end{align*}
\]

\( J(x) \) is clearly minimized by \( x^* \). Thus, the level of precaution chosen by the injurer in the solvent zone is socially optimal since the accident costs are fully internalized.

Let \( x_i \) denote the level of \( x \) that minimizes \( J_i(x) \); \( x_i \) is clearly equal to zero because \( J_i(x) \) is linear in \( x \). Since the injurer cannot curtail the probability of an accident by taking precaution, any precaution expenditure in the judgment proof zone is wasteful because it does not reduce the injurer’s expected liability while increasing his precaution costs. Hence, the injurer’s level of precaution, if he decides to be insolvent, will be \( x_i = 0 \). This proves the second claim.

The first claim is easily proven. The injurer will choose the optimal level of precaution, \( x^* \), if his total expenses at \( x^* \) are lower than his total expenses at \( x_i = 0 \), i.e. if \( J_i(0) \geq J(x^*) \), which can be rewritten as

(5) \[ t \geq h(x^*) + \frac{x^*}{p} \]

Otherwise, he will take no precaution. Note that Exp. (5) implies \( t > h(x^*) \); that is, it requires that the injurer is solvent at the optimal level of precaution. Figure 2 describes this result, where \( t^* \) denotes the smallest \( t \) that satisfies Exp. (5).
2.3 The joint-probability-magnitude model

In this model, the injurer’s precaution simultaneously reduces both the probability of the accident and the magnitude of the harm.

Let:

\[ p(x) = \text{the probability of an accident, } 0 < p(x) < 1, \quad p' < 0, \quad p'' > 0; \]

\[ h(x) = \text{the magnitude of the harm if an accident occurs, } 0 < h(x) \leq h_{\text{max}}, \quad h' < 0, \quad h'' > 0. \]

The social cost function is:

\[ S(x) = p(x)h(x) + x \]

Let \( x^* \) denote the level of precaution that minimizes the social cost function.

**Proposition 3**: In a joint probability-magnitude model, if the threshold is sufficiently high —

\[ t \geq p(x^*)h(x^*) + x^* - x_i \]

— the injurer takes optimal precaution. Otherwise, he takes underprecaution. The lower the threshold, the lower the level of precaution that the injurer takes when judgment proof.

We will employ the same algorithm as before. The injurer’s cost function is:

\[
J(x) = \begin{cases} 
  p(x)h(x) + x & \text{if } h(x) \leq t \\ 
  p(x)t + x & \text{if } h(x) > t
\end{cases}
\]

\[ J_r(x) = \begin{cases} 
  [\text{solvent zone}] \\
  [\text{judgment proof zone}]
\end{cases}
\]

\( J(x) \) is clearly minimized by \( x^* \). Thus, the level of precaution chosen by the injurer in the solvent zone is socially optimal since the accident costs are fully internalized. Let \( x_i \) denote the lowest point of the function \( J(x) \).

The injurer will choose the optimal level of precaution, \( x^* \), if his total expenses at \( x^* \) are lower than his total expenses at \( x_i \), that is if \( p(x_i)t + x_i \geq p(x^*)h(x^*) + x^* \), which can be rewritten as:

\[ t \geq \frac{p(x^*)h(x^*) + x^* - x_i}{p(x_i)} \]

Otherwise, he will take a lower than optimal level of precaution, \( x_i \). Moreover, \( x_i \) rises when the threshold \( t \) rises, which can be easily verified by applying the Implicit Function Theorem to the first order condition for \( J(x) \). However, \( x_i \) can be a solution of the injurer’s minimization problem only if it is lower than \( x^* \); that is, a higher level of precaution than optimal is never the result (see Appendix 2). Figure 3 depicts these results, where \( t^* \) denotes
the smallest $t$ that satisfies Exp. (8).

[Figure 3]

2.4 The separate-probability-magnitude model

In this model, the injurer can discriminate between the expenditure on reducing the probability of the accident and the expenditure on reducing the magnitude of the harm.

Let:

\[
\begin{align*}
p(s) &= \text{the probability of an accident, } 0 < p(s) < 1, p' < 0, p'' > 0; \\
h(z) &= \text{the magnitude of the harm if an accident occurs, } 0 < h(z) \leq h^\text{max}, h' < 0, h'' > 0.
\end{align*}
\]

Further, assume that the product $p(s)h(z)$ is a strictly convex function of $s$ and $z$. The social costs function is:

\begin{equation}
S(s, z) = p(s)h(z) + s + z
\end{equation}

Let $s^*$ and $z^*$ denote the levels of probability-reducing precaution and magnitude-reducing precaution respectively that minimize the social cost function.

**Proposition 4**: In a separate probability-magnitude model, the injurer takes optimal precaution if the threshold is sufficiently high — $t \geq [p(s^*)h(z^*) + s^* + z^* - s^*]/p(s^*)$. For intermediate levels of the threshold, the injurer takes no precaution with respect to the magnitude-reducing measure and overprecaution (or optimal precaution) with respect to the probability-reducing measure. For lower levels of the threshold, the injurer takes no precaution with respect to the magnitude-reducing measure and underprecaution with respect to the probability-reducing measure. The lower the threshold, the lower the probability-reducing precaution that the injurer takes when judgment proof.

The solution algorithm proposed above will again be employed. The injurer’s cost function is:

\begin{equation}
J(s, z) = \begin{cases} 
p(s)h(z) + s + z & \text{if } h(z) \leq t & \text{[solvent zone]} \\
p(s)t + s + z & \text{if } h(z) > t & \text{[judgment proof zone]}
\end{cases}
\end{equation}

$J(s, z)$ is minimized by $(s^*, z^*)$; thus, the levels of precautions chosen by the injurer are socially optimal.

Let $(s_t, z_t)$ denote the levels of precaution that minimize $J_t(s, z)$. As the function is linear in $z$, the injurer will take no magnitude-reducing precaution in the judgment proof zone ($z_t = 0$); the reason for this is the same as in the magnitude model. Conversely, $s_t$ might be lower than, equal to, or higher than $s^*$, which can be easily proven by looking at the first order conditions.
For \( J(s,z) \) to be minimized, the first order condition in relation to \( s \) is:

\[
(11) \quad p'(s^*) = \frac{-1}{h(z^*)}
\]

and, given \( z=0 \), the first order condition for \( J(s,z) \) to be minimized is:

\[
(12) \quad p'(s_i) = \frac{-1}{t}
\]

The injurer will under-invest in precaution to reduce the harm \((z_t=0)\) and under-invest in precaution to reduce the probability of an accident \((s_t<s^*)\) if \( t<h(z^*) \). On the contrary, if \( t<h(z^*) \), the injurer will under-invest in precaution to reduce the harm \((z_t=0)\) but will over-invest in precaution to reduce the probability of an accident \((s_t>s^*)\). If \( t=h(z^*) \), the level of probability-reducing precaution will be optimal \((s_t=s^*)\).

The injurer will choose the optimal level of precaution \((s^*,z^*)\) if his total expenses at that level are lower than at \((s_t,z_t)\); that is, the injurer chooses the optimal level of precaution if

\[
p(s^*)h(z^*)+s^*+z^*\leq p(s_t)t+s_t,
\]

which can be rewritten as:

\[
(13) \quad t \geq \frac{p(s^*)h(z^*)+s^*+z^*-s_t}{p(s_t)}
\]

Otherwise, he will take the inefficient levels \( s_t \) and \( z_t \). These results are summarized in table 2 below and in figure 4, where \( t^* \) denotes the smallest \( t \) that satisfies Exp. (13).

[Table 2]
[Figure 4]

### 3. Tort law solutions to the judgment proof problem

In the previous section, we showed that the effect of the judgment proof problem depends on the precaution technology. In this section, we will show that the optimal legal solution also depends on the precaution technology.

We will consider three corrective mechanisms that affect the size of the damages – punitive damages (overcompensation), undercompensation, and average compensation – and compare them with the situation analyzed so far, in which damages are perfectly compensatory. Our analysis is first made under a strict liability regime and then under the negligence rule.

#### 3.1 Overcompensation (punitive damages) and average compensation.

Punitive damages have been proposed to improve the judgment proof problem in a probability
model. The idea is that the injurer should pay more than the amount of the harm in those cases where the harm is lower than his assets and he is solvent in order to make up for those cases in which the harm is higher than his assets and he is insolvent.

In magnitude models, however, punitive damages worsen the judgment proof problem because they distort incentives in two ways. First, punitive damages distort the marginal decision of how much precaution to take by increasing the size of compensation for solvent injurers, and they thereby induce overprecaution. Second, punitive damages distort the inframarginal decision on whether to be solvent or insolvent and thus induce injurers to be insolvent. Likewise, average compensation removes the incentive to reduce harm as it makes the size of compensation independent of the harm actually caused.

In contrast, punitive damages may improve the problem in probability models. When the harm is lower than \( t \), the injurer can pay more than the harm. Punitive damages can, at most, be set so that the injurer pays his total assets \( t \) whenever the harm is lower than \( t \), but when the harm is higher than \( t \), the injurer pays \( t \) anyway since he is insolvent. Hence, the highest possible punitive damages make the injurer always pay \( t \). The incentive problem is only corrected if the threshold is higher than or equal to the expected harm, \( t \geq E(h) \). Figure 5 shows how punitive damages affect the damage payment of the injurer. The dotted zone depicts the maximum punitive damages applicable. To correct the incentives due to judgment proofness, the dotted area (maximum punitive damages) should be equal to or greater than the gray area in the upper right corner (uncompensated loss).

The same result however can be obtained by average compensation of the harm. Under average compensation the injurer does not pay the actual harm but instead pays the average harm of accidents of the same type, \( E(h) \). In other words, average compensation can only correct the incentive problem if the threshold is greater than or equal to the expected harm, \( t \geq E(h) \). These findings are formally proven in Appendix 4 and depicted in figure 6.

[Figure 5 and Figure 6]

Average compensation can thus correct the incentive problem to the same extent as punitive damages, but it has one major advantage: it requires less information. Average compensation only requires the court to have information on the average harm. It does not require information on the real harm in a specific case. Nor does it require ‘accurate’ compensation; indeed, average compensation can be seen as a form of inaccurate compensation. It also does not require information on the assets, \( t \), of an individual injurer.

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8 See Boyd and Ingberman (1994) and Lewis and Sappington (1999).

9 Accuracy in the determination of damages has been analyzed in a different contest from the judgment-proof problem by Kaplow (1994) and Kaplow and Shavell (1996).
With punitive damages, on the other hand, courts need information on the actual harm, the full distribution of the harm, and the individual threshold \( t \). In fact, punitive damages lead to a sort of *undeep-pocket rule*: the courts should apply a punitive measure only to those injurers who are potentially insolvent; if applied to solvent injurers, punitive damages would cause overprecaution. This mechanism charges the courts with a cumbersome task, as poor (and hence more likely to be insolvent) injurers have to pay over-compensatory damages in those instances in which they are actually solvent. Average compensation though can be applied in a very simple manner to both solvent and potentially insolvent injurers, without any distinction.\(^\text{10}\)

### 3.2 Negligence versus other forms of undercompensation

Undercompensation has also been proposed in the literature as a corrective mechanism for judgment proofness in magnitude models.\(^\text{11}\) The intuition is that, if the cost of being solvent is reduced through undercompensation, injurers will be more likely to take the optimal level of precaution than they are under perfect compensation.

The negligence rule is intrinsically an under-compensatory system since the accident costs caused by the injurer when he takes optimal precaution are externalized to the victim. Negligence, in fact, is the most effective form of undercompensation because it completely relieves non-negligent injurers of liability, whilst burdening negligent injurers with full liability.

Other forms of undercompensation based on a constant factor or on a lump sum reduction of the liability do not distinguish between the negligent and the non-negligent injurer but instead reward both of them with a sort of subsidy that is paid by the victim.

### 3.3 Negligence is superior to strict liability

The effects of a negligence rule have only been analyzed in probability models with the result that negligence reduces the judgment proof problem as compared to strict liability.\(^\text{12}\) This result also holds for the other three models, as proven in Appendix 3.

What is the underlying intuition? SHAVELL (1986, 45) attributed his findings to the

\(^{10}\) Average compensation may also reduce the rate of bankruptcy since small harms are overcompensated but large harms are undercompensated, and therefore, injurers will be found insolvent less frequently. If \( t \geq \mathbb{E}(h) \) for all potentially insolvent injurers, none of them will go bankrupt under an average compensation rule, while some of them would under perfect compensation. Punitive damages, on the other hand, increase the rate of bankruptcy compared to perfect compensation by increasing the number of cases in which the injurer has to pay more than he is able to pay. Boyd and Ingberman (1994) discussed the effect of punitive damages on the rate of bankruptcy but not the effect of average compensation.

\(^{11}\) See Boyd and Ingberman (1994).

‘sharpness of the incentives’; under a negligence rule, negligent injurers pay $p(x)h+x$, while non-negligent injurers pay only $x$. This creates a discontinuity in the injurer’s expenses function.

Another and perhaps more illuminating explanation might be that the negligence rule gives an implicit harm-subsidy in the amount of $p(x)h$ to non-negligent injurers. On the contrary, judgment proofness generates an implicit harm-subsidy to negligent injurers, which increases the more the level of precaution decreases. Under strict liability, the judgment proof subsidy operates alone and reduces the injurers’ incentives to take precaution. Under a negligence rule, the judgment proof subsidy to negligent injurers is counterbalanced by the negligence-subsidy to non-negligent injurers. This tends to rebalance the incentives towards optimal precaution.

Therefore, a negligence rule is less vulnerable to the judgment proof problem than a strict liability rule, but judgment proofness may still undermine the injurer’s incentives under a negligence rule. These findings would also hold true if we considered the interactions between the negligence criterion and the determination of causation.\(^{13}\)

4. Concluding remarks

We have discussed the effects of judgment proofness under four precaution technologies. In reality, an accident might be of a more complex type since the control of the injurer over the magnitude of the harm might be less than perfect and since the actual harm might be determined both by the injurer’s precaution and by stochastic variables. Such cases appear to be combinations of our basic probability model with one of the other three models.

The outcome might therefore be less sharp in reality than in our models. Nevertheless, our general conclusions about the possible solutions to the judgment proof problem might provide policy makers with useful indications about the best corrective mechanisms in these cases. Moreover, if the expenditure in precaution reduces the assets available for paying damage compensation, the injurer may actually be induced to take overprecaution, and the results derived in this article might not apply. This problem is analyzed in a separate study.\(^{14}\)

As for policymaking, our analysis suggests that a clear understanding of the accident precaution technology is crucial for assessing the effects of insolvency, liability caps, minimum capital requirements, and limited liability. Furthermore, with respect to solutions, our findings seem to strengthen the case for the good old negligence rule.

\(^{13}\) See Grady (1983) and Kahan (1989).

\(^{14}\) See Dari Mattiacci and De Geest (2003).
Appendix 1 : The algorithm solution employed in the magnitude models

Exp. (5) guarantees that the restrictions imposed on the equations of Exp. (4) are always satisfied by the solution. In particular, if the solution of the injurer’s minimization problem is $x^*$, then it can always be verified that the injurer is solvent at $x^*$, i.e. $h(x^*) \leq t$. Indeed, the opposite is impossible.

Assuming that $h(x^*) > t$, then $ph(x^*) + x^* > pt + x^*$. However, by the definition of $x^*$, we can write $pt + x^* > pt + x^*$. Therefore, $ph(x^*) + x^* > pt + x^*$ would also be true, but this contradicts our premise that $x^*$ is the solution since this would imply that $x_r$ is the solution.

Thus, if $x^*$ is the solution, then $h(x^*) \leq t$ must be satisfied. In a similar way, it is possible to verify that if $x_r$ is the solution, then the injurer is insolvent at $x_r$. Therefore, the simple algorithm solution proposed always provides a valid solution to the injurer’s minimization problem. The same applies to the joint-probability-magnitude model and the separate-probability-magnitude model.

Appendix 2 : Proof that $x_t < x^*$ in the joint-probability-magnitude model

Assume the opposite is true, i.e. assume $x_t \geq x^*$; in order for $x_t$ to be a solution of the injurer’s minimization problem, the injurer must be insolvent at $x_t$, i.e. the condition $h(x_t) > t$ must be satisfied. Note that if the injurer were solvent at $x_t$, then the solution of the problem would necessarily be $x^*$, see Appendix 1 on this point. Consequently, $h(x^*) > t$ is also true since $h$ increases as $x$ decreases. Let us now apply these findings to the comparison between the first order conditions for $J(x)$ and $J_t(x)$. The former is $p' (x^* ) h(x^*) + p (x^*) h' (x^*) = -1$; the latter yields $p' (x_t ) t = -1$. Thus we can write $p' (x_t ) h(x_t ) + p (x_t ) h' (x_t ) = p' (x_t ) t$. Since we have noticed that $h(x^*)$ is greater than $t$, and as all the addenda are negative quantities, then the equality is satisfied only if $p' (x_t ) < p' (x^*)$, which implies that $x_t < x^*$. This in turn contradicts the premise that $x_t \geq x^*$.

Moreover, note that even if $t = t^*$, $x_t$ is still smaller than $x^*$, which justifies the discontinuity in figure 3. This point is easily proven. In fact, the first order condition that yields $x_t$ is $p' (x_t ) = -1/t$. Substituting $t^*$ therein, we obtain:

\[ p' (x_t ) = \frac{- p (x_t )}{p (x^*) h(x^*) + x^* - x_t} \]

(A.1)

If $x_t = x^*$ when $t = t^*$, Exp. (A.1) may be rewritten as $p' (x^*) = -1/h(x^*)$. Nevertheless the latter cannot be true as $p' (x^*)$ must satisfy the first order condition for $x^*$, which is different from $-1/h(x^*)$. Hence, at $t^*$, $x_t$ is smaller than $x^*$. Note also that the difference between $x_t$ and $x^*$ at $t^*$
depends on the first derivative of $h(x)$, which in turn can be interpreted as the measure of the injurer’s ability to reduce the magnitude of the harm. When $h'$ approaches zero (the injurer has little influence on $h$), $x_i$ becomes closer to $x^*$ at $t^*$, and the joint-probability-magnitude model becomes similar to a simple probability model. On the contrary, when the absolute value of $h'$ increases (the injurer has greater influence on $h$), $x_i$ significantly diverges from $x^*$ and the model becomes more similar to a simple magnitude model.

Appendix 3: Negligence

a) Probability model. Under strict liability, a judgment proof problem reduces the injurer’s incentive if $t< h_{\text{max}}$. On the contrary, if the rule in force is negligence, $t< h_{\text{max}}$ might not always result in underprecaution. In fact a non-negligence injurer pays $J(x^*)=x^*$, while a negligent (judgment proof) injurer pays $J(x_i)=p(x_i)+x_i$. The injurer takes optimal precaution if $J_n(x^*) \leq J_t(x_i)$, which can be rewritten as:

$$t \geq \frac{x^*-x_i}{p(x_i)} \quad \text{(A.2)}$$

Negligence reduces the effect of the judgment proof problem in some cases in which judgment proofness would have undermined the injurer’s incentives under strict liability – that is, when $t< h_{\text{max}}$. The proof is simple. When $t$ approaches $h_{\text{max}}$ (that is, when a judgment proof problem arises under strict liability), $x_i$ also approaches $x^*$, and as a result, the right-hand side of (A.2) approaches zero and is thus smaller than some $t$. So, no judgment proof problem arises under negligence.

b) Magnitude model. Under a magnitude model, the injurer decides not to be potentially judgment proof (and to take optimal precaution) if $J_n(x^*)=x^* \leq pt=J_t(x_i)$, which can be written as:

$$t \geq \frac{x^*}{p} \quad \text{(A.3)}$$

In this case too, negligence prevents the effect of the judgment proof problem for some level of $t$ that would have undermined the injurer’s incentive under strict liability. To see why, compare Exp. (A.3) with Exp. (5).

c) Joint-probability-magnitude model. The logic is the same under a joint-probability-magnitude model. The injurer decides not to be potentially judgment proof (and takes optimal
precaution) if \( J_n(x^*) = x^* \leq p(x) t + x_t = J_t(x_t) \), which may be written as:

\[
(A.4) \quad t \geq \frac{x^* - x_t}{p(x_t)}
\]

In this case, compare Exp. (A.4) with Exp. (8).

d) *Separate-probability-magnitude model.* Under a separate-probability-magnitude model, the injurer decides not to be potentially judgment proof (and takes optimal precaution) if \( J_n(s^*, z^*) = s^* + z^* \leq p(s_t) t + s_t = J_t(s_t) \), which may be written as:

\[
(A.5) \quad t \geq \frac{s^* + z^* - s_t}{p(s_t)}
\]

In this case, compare Exp. (A.5) with Exp. (13).

**Appendix 4: Punitive damages versus average compensation in probability models**

Punitive damages, \( D(h) \), will completely correct the judgment-proof problem only if the expected compensation is increased up to the average harm:

\[
(A.6) \quad E_t(h) + D(h) = E(h).
\]

\( E_t(h) + D(h) < E(h) \) would lead to underprecaution, while \( E_t(h) + D(h) > E(h) \) would lead to overprecaution. When punitive damages are set according to Exp. (A.6), the potentially-judgment-proof injurer’s cost function is again \( p(x) E(h) + x \), which is minimized by \( x^* \) (the optimal level of precaution). The underprecaution problem is solved completely.

Nevertheless, punitive damages are subject to a limit. The maximum amount of punitive damages that can be applied to an injurer is in fact determined by the difference between the threshold, \( t \), and the harm, \( h \), that actually occurred in the accident if \( h \) is lower than \( t \). Ex ante, the maximum punitive damages an injurer may expect to be charged with is:

\[
(A.7) \quad D_m(h) = \int_0^t f(h)(t - h)dh
\]

The injurer takes optimal precaution only if \( E_t(h) + D_m(h) \geq E(h) \), which can be rewritten as

\[
\int_0^t f(h)dh + \int_0^{h_{max}} f(h)tdh + \int_0^t f(h)(t - h)dh \geq E(h) \quad \text{or}
\]

\[
(A.8) \quad t \geq E(h)
\]

Otherwise, he takes a sub-optimal level of precaution, which is still higher than the level of
precaution the insolvent injurer takes under compensatory damages (since he internalizes $t$ instead of $E_t(h)<t$).

Under average compensation, the injurer always pays $E(h)$. If the injurer can pay the average harm, then average compensation completely solves the judgment-proof problem. Thus, the limit for average compensation being a perfect solution to the problem is again $t \geq E(h)$, as in the case of punitive damages.

**Appendix 5: Combining average compensation and negligence in probability models**

If both average compensation and negligence are implemented and if $t \geq E(h)$, then the injurer’s cost function is:

\[
J_t(x) = \begin{cases} \frac{p(x)E(h) + x}{2} & \text{if } x < x^* \\ x & \text{if } x \geq x^* \end{cases}
\]

Thus, the injurer takes optimal precaution $x^*$. If $t < E(h)$, the injurer’s cost function is:

\[
J_t(x) = \begin{cases} \frac{p(x)t + x}{2} & \text{if } x < x^* \\ x & \text{if } x \geq x^* \end{cases}
\]

The injurer takes $x^* \geq x_a$ if $x^* \geq p(x_a)t + x_a$, where $x_a$ is the level of precaution that minimizes $p(x)t + x$. Otherwise, he takes $x_a$. If average compensation is used alone, the injurer always takes $x_a$. If negligence is used alone, Exp. (A.10) becomes:

\[
J_t(x) = \begin{cases} \frac{p(x)E_t(h) + x}{2} & \text{if } x < x^* \\ x & \text{if } x \geq x^* \end{cases}
\]

The injurer takes $x^*$ if $x^* \geq p(x)E_t(h)+x_t$, where $x_t$ is the level of precaution that minimizes $p(x)E_t(h)+x_t$; he takes $x_t$, otherwise. It is important to notice that since $x_a > x_t$, as $t > E_t(h)$, the level of precaution taken by negligent injurers when negligence is combined with average compensation is higher than when negligence is used alone. Moreover, as clearly $p(x_a)t + x_a > p(x_t)E_t(h)+x_t$, optimal precaution arises more often when average compensation and negligence are combined.

**References**


**Figures**

**Table 1**
Summary of the results

<table>
<thead>
<tr>
<th>Precaution technology</th>
<th>Level of precaution</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability model</td>
<td>Underprecaution</td>
<td>Negligence with average compensation</td>
</tr>
<tr>
<td>Magnitude model</td>
<td>No precaution <em>or</em> optimal precaution</td>
<td>Negligence with perfectly compensatory damages</td>
</tr>
<tr>
<td>Joint-probability-magnitude model</td>
<td>Underprecaution <em>or</em> optimal precaution</td>
<td>Negligence with perfectly compensatory damages</td>
</tr>
<tr>
<td>Separate-probability-magnitude model</td>
<td>No precaution with respect to magnitude and under-, optimal, or overprecaution with respect to probability <em>or</em> optimal precaution with respect to both</td>
<td>Negligence with perfectly compensatory damages</td>
</tr>
</tbody>
</table>

**Table 2**
Precaution in separate probability-magnitude models

<table>
<thead>
<tr>
<th>Threshold</th>
<th>Probability-reducing precaution</th>
<th>Magnitude-reducing precaution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t &lt; h(z^*)$</td>
<td>$s_t &lt; s^*$</td>
<td>$z_t = 0$</td>
</tr>
<tr>
<td>$t = h(z^*)$</td>
<td>$s_t = s^*$</td>
<td>$z_t = 0$</td>
</tr>
<tr>
<td>$h(z^<em>) &lt; t &lt; t^</em>$</td>
<td>$s_t &gt; s^*$</td>
<td>$z_t = 0$</td>
</tr>
<tr>
<td>$t \geq t^*$</td>
<td>$s_t = s^*$</td>
<td>$z_t = 0$</td>
</tr>
</tbody>
</table>
Figure 1
Level of precaution in the probability model

Figure 2
Level of precaution in the magnitude model
Figure 3
Level of precaution in the joint-probability-magnitude model

Figure 4
Level of precaution in the separate-probability-magnitude model
(Note that $s^*$ ought not necessarily to be lower than $z^*$)
Figure 5
Punitive damages and average compensation in the probability model

\[ D_t(h) = E(h) \]

\[ h_{\text{max}} \]

Figure 6
Level of precaution in the probability model with average compensation